Research of Properties of Surface of Detail

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Abstract

The article is devoted to problems associated with geometric precision production cylindrical parts on lathes with systems of CNC. The possibility of a high view of cylindrical shapes using a complex adaptive system working process.

Basics

In modern metal processing there are many problems associated with the manufacture of parts for which it is important not so much to get high accuracy as the size of perfect geometries. This problem sometimes does not allow to get the details of precision geometry seemingly technical in those processes where it should be achieved without much effort. This problem is an older bug of disconnected system MATD (machine - accessories - tools - detail) that causes a lot of complications. The nature of these complications is more mathematical problems than technical associated with direct binding into one system MATD order to get a closed technological circle. Most of these problems are solved directly, it is the application of interim results measurement control devices that automatically shifted to the processing tool. From here is actuality of problem and proper raising of task and decision. The offered problem requires the next raising of task and ways of its decision:

- determination of mathematical apparatus is in relation to a task from the receipt of veritable form of treatment of the object;
- consideration of situations which arise up in the process of measuring as on idealizing situations so with the gradual passing to the real;
- determination of kinematics of optimum motion from point of balance between measuring time and minimum necessary information enough for the decision of problems correction of detail form;
- a mathematical apparatus of correction form of detail is in order to receive geometrically regular shapes;
- apparatus providing of construction technological process measuring of form and its correction.

The article is third on offered issue in relation to lathe treatment, and that is why in it the problem phenomena will be examined related to determination of the generalized form of detail. In a general plan theoretical subsoil was stopped up in process [1]. Partially the problems were examined in relation to milling treatment in process [2] where the form of detail was examined by a milling cutter.

This problem was also partly considered in the previous articles. So in the article [3] was researched the possibility of determination of the form of cylindrical body in the planes *X* and *Y*. In the article [4] was researched the possible methods of determination of longitudinal geometry of cylinder details with foot-pace motion of instrument.

For valuable consideration of the task it is necessary to consider the followings factors of influence on end-point result of measuring:

- to properties of surface as object of research with the purpose of determination of parameters with the most informing;
- motion of instrument with measuring in a spiral method;
- the information which possibility to get at such method of instrument motion.

Motion of measuring instrument.

In obedience to the put task in the article, research of geometry of surface takes a place after the spiral trajectory of motion. An in itself cutting instrument can not move above the surface of cylinder form, because this motion is difficult in the basis. In concrete case the top of instrument moves along a detail with the serve of s. Direction of motion as a rule from a right to the left, if there are not the special technological terms. At the same time a detail is revolved around a landmark X . Complex motion is thus created which possibility to engulf researches considerable part of surface of detail is at.

For that, exactly to present possibilities of such motion will consider him mathematical expression. From all of variants of consideration of curve in space [6] most optimum from point of the put task is its task in a self-reactance kind:

$$x = x(t),\ y = y(t),\ z = z(t), \qquad (1)$$

where $\alpha \le t \le \beta(-\infty \le \alpha, \beta \le +\infty)$.

As a local element of curve in space is length of arc of s between two points of Mcode and D, that answer the value of parameter of to and t, it will be evened:

$$s = \int_{t_0}^{t} ds\ , \qquad (2)$$

where $ds = \sqrt{(\dot{x}(t)\;^2 + (\dot{y}(t)\;^2 + (\dot{z}(t)\;^2}\,dt$.

In the case of jigging of point of Mcode in quality new the parameter of curve it is possible to enter s (parameter of arc). In the case when s positive, Mcode follows D (as an index by t – positive direction). Then expressions (1) will get the following kind:

$$x = x(s),\; y = y(s),\; z = z\,(s). \qquad (3)$$

Such trajectory of motion as well as every curve in space is characterized three basic parameters. These parameters is: K is curvature of curve; $R = 1/K$ it is a radius of curvature; T_c – twisting trajectory on a curve. In accordance with it:

$$K = \sqrt{\left(\frac{d^2x(s)}{ds^2}\right)^2 + \left(\frac{d^2y(s)}{ds^2}\right)^2 + \left(\frac{d^2z(s)}{ds^2}\right)^2} \qquad (4)$$

Accordingly T_c

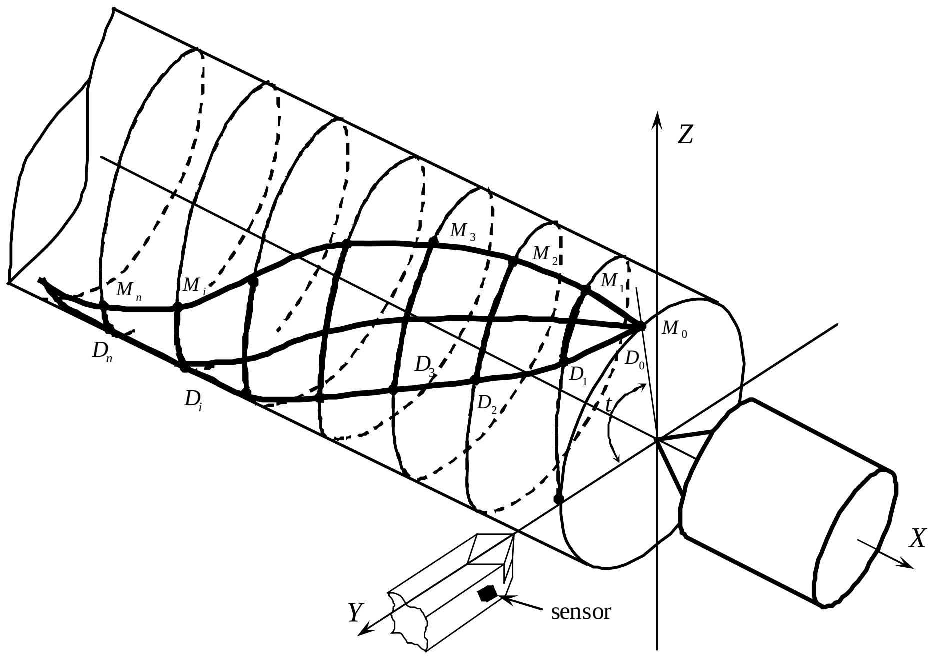
$$T_c = R^2\left(\frac{d\mathbf{r}(s)}{ds}, \frac{d^2\mathbf{r}(s)}{ds^2}, \frac{d^3\mathbf{r}(s)}{ds^3}\right) = R^2\,\frac{\begin{matrix}\dot{x} & \dot{y} & \dot{z}\\ \ddot{x} & \ddot{y} & \ddot{z}\\ \dddot{x} & \dddot{y} & \dddot{z}\end{matrix}}{(\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)} \qquad (5)$$

where $\dot{x}, \ddot{x}, \dddot{x}$ there is the first, third flexion for s, and others like that.

Expressions (4) and (5) is not very much comfortable, that is why the followings dependences are more frequent utillized derivates from previous:

$$K^2 = \frac{1}{R^2} = \frac{(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)(\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2) - (\dot{x}\,\ddot{x} + \dot{y}\,\ddot{y} + \dot{z}\,\ddot{z})^2}{(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^3}, \qquad (6)$$

Using the higher noted mathematical dependences will consider the trajectory of motion of instrument above the surface of detail (Pic.1)



Pic.1. Trajectory of motion of instrument above the surface of detail in the mode of measuring.

At such method of motion will have the followings parameters of trajectory:

$z = a\cos t,\ y = a\sin t,\ x = bt,\ a>0,$ (7)

where b>0 is a right spiral line, (Pic.2) or b<0 is the left spiral line.

Replacing a parameter to length of arc get:

$$s = \int_0^t \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt = t\sqrt{a^2 + b^2} \quad (8)$$

from where

$$z = a\cos\frac{s}{\sqrt{a^2+b^2}},\ y = a\sin\frac{s}{\sqrt{a^2+b^2}},\ x = \frac{bs}{\sqrt{a^2+b^2}}. \quad (9)$$

Thus, curvature

$$K=\frac{1}{R}=\sqrt{\left(\frac{d^2x}{ds^2}\right)^2+\left(\frac{d^2y}{ds^2}\right)^2+\left(\frac{d^2z}{ds^2}\right)^2}=\frac{a}{a^2+b^2} \qquad (10)$$

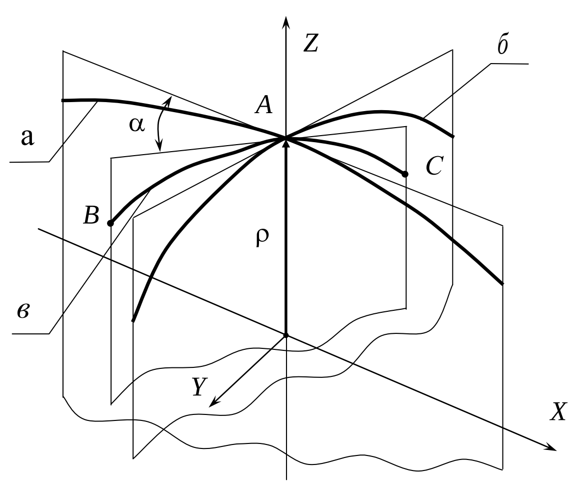
will have permanent character. Concordantly (5) and (6) twisting trajectory on a curve will make:

$$T=\left(\frac{a^2+b^2}{a}\right)^2\frac{\begin{matrix} b & a\cos t & -a\sin t \\ 0 & -a\sin t & -a\cos \\ 0 & -a\cos t & a\sin t\end{matrix}}{\left[(-a\sin t)^2+(a\cos t)^2+b^2\right]^3}=\frac{b}{a^2+b^2}, \qquad (11)$$

and also will be permanent within the limits of object of measurement.

Research properties of detail surface

We are considered the properties of surface in combination with the properties of curves that located on this surface to find out problems which arise up at research of it. According to metrical properties of surface (pic.2) the first quadratic form turns out as follows.



Pic. 2. Curves are located on the surface: a - is a curve of longitudinal cut in the plane *ZOX;* б - is a curve of transversal cut in the plane *ZOY*; *в* – is a curve BAC for the trajectories of instrument motion; α - corner between planes of the cut.

If a surface is set in a vectorial form in this case it will be: **r=r**(*u, v*) and accordingly *u = u(t), v = v(t)*, **r** = **r(t) = r**(*u(t)v(t))* shows by itself a curve on the surface. Then fair dependence:

$$\frac{d\mathbf{r}}{dt} = \mathbf{r}'_u \frac{du}{dt} + \mathbf{r}'_v \frac{dv}{dt}. \qquad (12)$$

The differential of length of arc, concordantly (2), will look like:

$$ds^2=(d\mathbf{r})^2=Edu^2+2Fdudv+Gdv^2, \qquad (13)$$

where

$$E = \left(\frac{\partial \mathbf{r}}{du}\right)^2 = \left(\frac{\partial x}{du}\right)^2 + \left(\frac{\partial y}{du}\right)^2 + \left(\frac{\partial z}{du}\right)^2,$$

$$G = \left(\frac{\partial \mathbf{r}}{\partial v}\right)^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2,$$

$$F = \frac{\partial \mathbf{r}}{\partial u}\frac{\partial \mathbf{r}}{\partial v} = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}.$$

In case if a surface is set as $x = x(z,y)$, then:

$$E = 1 + \left(\frac{\partial x}{\partial z}\right)^2, F = \frac{\partial x}{\partial z}\frac{\partial x}{\partial y}, G = 1 + \left(\frac{\partial x}{\partial y}\right)^2. \qquad (14)$$

The first quadratic form determines all of the metrical properties of surface. Length of arc BAC curve, if it is set as **r** = **r**(*u(t)*,v(t)) on the surface between points which answer the value of parameter *of* t_0 and *t* will be:

$$s = \int_{t_0}^{t} ds = \int_{t_0}^{t} \sqrt{E\left(\frac{du}{dt}\right)^2 + 2F\frac{du}{dt}\frac{dv}{dt} + G\left(\frac{dv}{dt}\right)^2}\, dt \qquad (15)$$

We can define the corner between two curves on the surface if $r = r(u_1(t),v_1(t))$, and $r = r(u_2(t),v_2(t))$ are two curves on the surface $r = r(u,v)$ which intersect in the point of *A*. From here the corner of crossing α (corner between positive directions of tangent in a point *A*) calculated on a formula:

$$\cos\alpha = \frac{E\,\dot u_1\,\dot u_2 + F(\dot u_1\,\dot v_1) + G\,\dot v_1\,\dot v_2}{\sqrt{E\dot u_1^2 + 2F\dot u_1\dot v_1 + G\dot v_1^2}\sqrt{E\dot u_2^2 + 2F\dot u_2\dot v_2 + G\dot v_2^2}} \qquad (16)$$

where $\dot{u}_1$ and $\dot{u}_2$ - accordingly the first derivative from $u_1(t)$ and $u_2(t)$ at the value of parameter which answers to the point *A,* and others like that.

If to take advantage of the second quadratic form of surface:

$$-d\mathbf{N}d\mathbf{r} = Ldu^2 + 2Mdudv + Ndv^2, \qquad (17)$$

that gives possibility to define properties of curvature of the surface, then for the second quadratic form of surface will be correct the following equalizations:

$$L = \frac{l}{\sqrt{EG-F^2}}, N = \frac{n}{\sqrt{EG-F^2}}, M = \frac{m}{\sqrt{EG-F^2}}, \qquad (18)$$

where

$$l = \begin{vmatrix} \frac{\partial^2 x}{\partial u^2} & \frac{\partial^2 y}{\partial u^2} & \frac{\partial^2 z}{du^2} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}, \; m = \begin{vmatrix} \frac{\partial^2 x}{\partial u \partial v} & \frac{\partial^2 y}{\partial u \partial v} & \frac{\partial^2 z}{du \partial v} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}, \; n = \begin{vmatrix} \frac{\partial^2 x}{\partial v^2} & \frac{\partial^2 y}{\partial v^2} & \frac{\partial^2 z}{\partial v^2} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}. \qquad (19)$$

For main curvature in the fixed point of surface *A* it is always possible to choose the same cartesian system of coordinates *X, Y, Z*, for which beginning of coordinates belongs *A,* and the plane *XOY* coincides with a tangent plane which passes through *A*. In this system of coordinates a surface (within the limits of point *A*) can be presented as $x = x(z,y)$, where dependence is executed:

$$x(0,0) = \frac{\partial x(0,0)}{\partial z} = \frac{\partial x(0,0)}{\partial y} = 0. \qquad (20)$$

The proper trihedral which accompanies surface in a point *A* will be consists of three unit vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{N} = \boldsymbol{e}_1 \times \boldsymbol{e}_2$ which are directed near coordinate axes. Equation by the formula of Teylor within the limits of point A will look like:

$$\tilde{o} = \frac{1}{2}\frac{\partial^2 x(0,0)}{\partial z^2} z^2 + \frac{\partial^2 x(0,0)}{\partial z \partial y} zy + \frac{1}{2}\frac{\partial^2 x(0,0)}{\partial y^2} y^2 + ... \quad (21)$$

If to take advantage of turn of the cartesian system of coordinates around to the axis *X* it is possible to have:

$$x = \frac{1}{2}(k_1 z^2 + k_2 y^2) + ... \quad (22)$$

where sizes k_1, k_2 is main curvatures of surface, and

$$R_1 = \frac{1}{k_1}, \ R_2 = \frac{1}{k_2} \quad (23)$$

it is the main radiuses of curvature.

Size

$K = k_1 k_2$ (24)

it is Gaus curvature, where

$$H = \frac{1}{2}(k_1 + k_2) \quad (25)$$

it is middle curvature in a point *A*..

In the appointed system of coordinates, where a surface can be imaginable in a kind 22, the quadratic forms in a point *A* have the simplified kind:

$$(dr)^2 = dz^2 + dy^2, \ -d\mathbf{N}d\mathbf{r} = k_1 dz^2 + k_2 dy^2. \quad (26)$$

For the arbitrary system of coordinates on surfaces we have equalities:

$$K = \frac{LN - M^2}{EG - F^2}, H = \frac{LG - 2FM = EN}{2(EG - F^2)} \quad (27)$$

where main curvatures k_1, k_2 is a root of quadratic equation $k^2 - 2Hk + K = 0$.

Substantial advantage of this use in differential form consists in that we know these sizes in the system of coordinates and know their conduct at transformation. And so it is possible to get expression of these sizes in the arbitrary system of coordinates. For main curvature at the change of orientation we can change only a sign, so *K* in transition to the arbitrary system does not change. To find expression for *K* in the arbitrary system it is necessary only to build scalar, that in the certain system of coordinates coincides with k_1 and k_2. So like that we got the scalar.

Classification of points of surface (22) allows to define the type of surface within the limits of point A (in the certain system of coordinates).

If to cross a surface with a plane, which passes through normal to the surface in a the point *A* we will get a normal crossing in the point *A*. In this case it is always will exist two mutually perpendicular directions in which curvature of the proper normal cuts in the point *A* will equal main curvatures k_1, k_2. These directions are equivalent to directions of axes in the certain system of coordinates and will be equivalent to directions of main curvature, and the proper crossings - to the main normal crossings of surface. If a secant plane will form a corner α with the axis e_1 then for curvature of normal crossing k_N in the point *A* will be correct this dependence (formula of Euler):

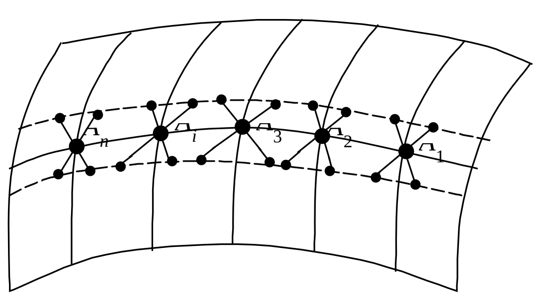
$$k_N = k_1 \cos^2 \alpha + k_2 \sin^2 \alpha \quad (28)$$

Accordingly in directions of main curvatures the curvature of the normal crossing k_N takes extreme (maximal and minimum) values. They are equivalent to main curvatures k_1, k_2.

The connection between coordinates of touch of instrument with a detail and its form (method)

As evidently from the higher resulted mathematical consideration of the surface properties it is possible to make a few methods, in relation to determination of expression of description of detail surface, but in basis of all of these methods it must be the matrix field of coordinates of surface. In an order to present which one coordinates of surface we have possibility get we will consider what takes a place on the concretely select area of surface during registration of coordinate.

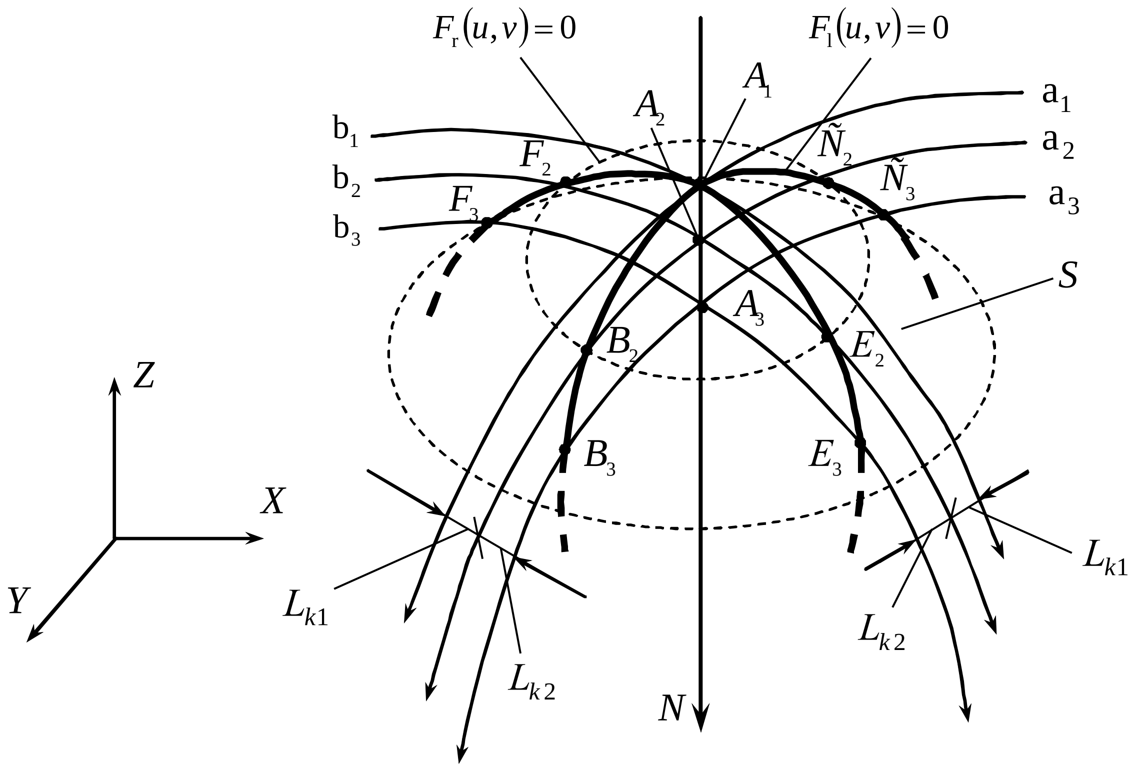
Such situation arises up as a result of two meetings motions of instrument and is explanation that takes a place at definition of coordinates of surface for picture 3.



Pic. 3. There is a location of the special informative points on the surface of the detail.

By such method of the motion foremost we determined the coordinate of point A, which is maximally remote from the axis X after a static coordinate. Besides during the proper synchronization direct and to reverse motion (left-side and right-side pic.4.) is confirmed coordinate of the point A, with minimum divergence which is determined the sensitiveness of the touch system. This is take place over the trajectories of b>0 and b<0 concordantly z = acost, in = asint, x = bt, a>0, where b>0 is a right spiral line, or b<0 is the left spiral line $(a_1, a_2, a_3$ і $б_1, б_2, б_3)$

At a next step to the center of receipt of detail the radius of motion diminishes on L_{k1} and the system CNC registers the coordinates of surface C_2 and B_2 for left-side motion and F_2 and E_2 for right-side. The got coordinates of five points surface *XOY* are ponderable enough for the informative point of view.



Pic. 4. The trajectories of instrument motion are for the receipt of coordinates of surface of detail: a_1, a_2, a_3 of – left-side trajectory of motion and b_1, b_2, b_3 are right-side trajectories of motion.

The source of this problem is that such amount of points of co-ordinates does not determine the type of surface. There is possibility to conduct a few surfaces of different type in space through five points. It is visible from mathematical dependences (19, 20, 21, 24, 25). It is therefore necessary in the certain coordinate x to do yet one touch with the purpose of determination of coordinates of surface. Working another motion (L_{k2}) the system CNC gets the duty coordinates C_3 and B_3 in left-side motion and F_3 and E_3 at right-side motion. In such case general amount of coordinates is nine, that gives possibility with high authenticity to define the form of surface after the higher mentioned dependences. At such method of motion there are two important situations that closely associated with method of getting coordinates and which need to be perceived as clever limitation.

At first it concern the feed of instrument on the coordinate X and speeds of rotation $\omega_Ä$. If $\omega_Ä$ infinitely grows at the stable serve of instrument S_i coordinates of points B_2, B_3, C_2, C_3 from one side unlimited approach the coordinates of points E_2, E_3, F_2, F_3 and in endlessness meet in one unit in the plane to the cut of detail with the coordinates of point A. So here will be a case of determination of form considered in a cut [3]. Accordingly in opposite case when $\omega_Ä \to 0$ and $S_i \to \infty$ we will have a case considered in [4]. Thus coordinates of points , B_3, C_2, C_3 та E_2, E_3, F_2, F_3 will be unlimited to approach one to one and to the planes *ZOX*.

Secondly with the purpose of getting maximal speed at determination of form is desirable upon termination of measuring of every pair of coordinates (B_2-C_2, B_3-C_3, E_2-E_3 and others like that) to work off a step in a side the axis of rotation. This problem is especially technical and touches the inertance of the motive system of machine-tool which is ponderable limit on speed of measuring process.

If to select the actual speed of measuring of the form it is necessary to pay a regard to the second point of limitations because first is far less in it's inertance than inertance of mechanical motions.

From consideration of mathematical properties of surface it is possible to draw a conclusion about possibility of use the few methods in relation to determination of it general view. The offered method of instrument motion on a spiral trajectory is researched from point of informative possibilities. We defined certainly basic limitations and optimum of measuring criteria.

On the basis of mathematical analysis it was conducted the foundations of doing system, that proposed tapping tool to the detail. A flow chart and touch the device registration unit was designed to work as part of CNC. It was developed flowchart of registration touching device and the unit was designed to work as part of CNC.

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