ESTIMATION OF DEFECTS GEOMETRIC PARAMETERS WITH A THERMAL METHOD

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ABSTRACT. The problem of estimation of flaws' parameters has been realized in two stages. At the first stage, it has been estimated relationship between temperature difference of a heated sample's surface and geometrical parameters of the flaw. For this purpose we have solved a direct heat conduction problem for various combination of the geometrical sizes of the flaw. At the second stage, we have solved an inverse heat conduction problem using the H - infinity method of identification. The results have shown good convergence to real parameters.

INTRODUCTION

Estimation of defects geometric parameters, which is based on measurement of the object surface temperature, is the inverse problem of the heat conduction. It is complicated to identify the internal geometric and thermal characteristics using results of temperature measurements because of the effects of temperature smoothing and temperature signal delay.

There are two method groups of solution of the thermal testing inverse problem [1]. The first one is indirect, based on using a modification of iterative solving of the direct thermal testing problem and convergence of the experimental and calculated data. The second method group is based on approximation of the geometric distribution of thermal characteristics in defective structures by the smooth functions and following substitution of the initial differential equation for a simpler one.

The disadvantages of the mentioned methods are working hours and impossibility to use them for systems working in real time. Thus, the development of the most effective methods is a high-priority task.

PROBLEM STATEMENT

Consider the problem of testing of a one-layer plate M_1 (object of testing) with a cavity M_2 (Figure1). The cavity thermal characteristics are different from the basic material ones. Then, the mathematical model of a heat process is described by the two-dimensional nonstationary equation of heat conduction inside of the regions M_i ($i = 1,2$):

$$c_i \rho_i \frac{\partial T}{\partial t} = \lambda_i \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) , \qquad (1)$$

where T – is temperature; t – is current time; x, y – are space coordinates of the analyzed positions of the i-th region $(x, y) \in \sum_{i=1}^{2} M_i$; c_i , ρ_i , λ_i – are heat capacity, density and heat conduction factor of the i-th region accordingly.

There is the condition of thermal interface on the border of the regions M_1 and M_2 :

$$\lambda_1 \frac{\partial T}{\partial x} \Big|_{x=x_1-0} = \lambda_2 \frac{\partial T}{\partial x} \Big|_{x=x_1+0} , \qquad (2)$$

$$\lambda_1 \frac{\partial T}{\partial x} \Big|_{x=x_2+0} = \lambda_2 \frac{\partial T}{\partial x} \Big|_{x=x_2-0} ,$$

$$T \Big|_{x=x_1-0} = T \Big|_{x=x_1+0} , \quad T \Big|_{x=x_2-0} = T \Big|_{x=x_2+0} , \quad 0 \le x \le L_x ,$$

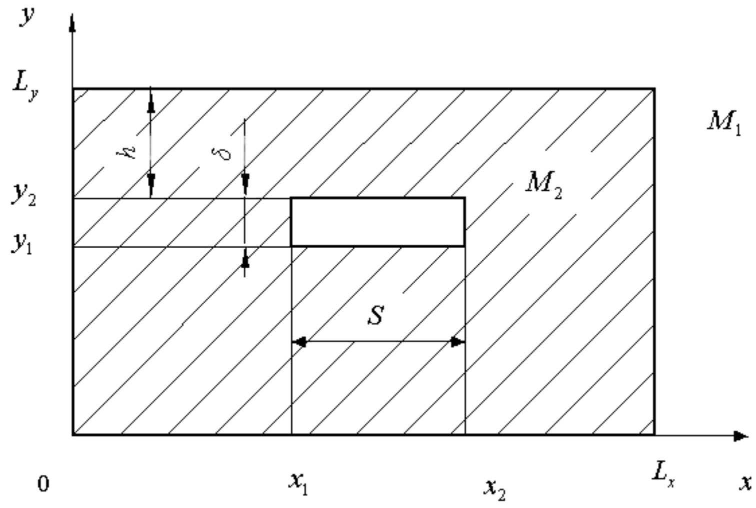

Figure 1. The sectional view of the one-layer sample with a defect.

$$\lambda_1 \frac{\partial T}{\partial y} \Big|_{y=y_1-0} = \lambda_2 \frac{\partial T}{\partial y} \Big|_{y=y_1+0}, \quad (3)$$

$$\lambda_1 \frac{\partial T}{\partial y} \Big|_{y=y_2+0} = \lambda_2 \frac{\partial T}{\partial y} \Big|_{y=y_2-0},$$

$$T \Big|_{y=y_1-0} = T \Big|_{y=y_1+0}, \quad T \Big|_{y=y_2-0} = T \Big|_{y=y_2+0}, \quad 0 \le y \le L_y, \quad t > 0 .$$

The boundary conditions on the $y = L_y$ surface are:

$$\lambda_1 \frac{\partial T}{\partial y} \Big|_{y=L_Y} = Q, \quad (4)$$

where Q – is thermal flow from a heater.

The surfaces of $x = 0$, L_x and $y = 0$ are in free heat exchange with the environment under Newton's law:

$$\lambda_1 \frac{\partial T}{\partial x} \Big|_{x=0,L_X} = \alpha(T - T_{env}), \quad 0 \le y \le L_y, \quad (5)$$

$$\lambda_1 \frac{\partial T}{\partial y} \Big|_{y=0} = \alpha(T - T_{env}), \quad 0 \le x \le L_x,$$

where α – is heat exchange factor, T_{env} – is temperature of the environment.

At the initial phase of the experiment the temperature of the region M_i ($i = 1,2$) is equal $T \big|_{t=0} = T_{env}$. The problem is to find the defect parameters by measuring the surface temperature of the sample.

SOLUTION

The main defect parameters, which influence on the space-time structure of a thermal field, are its location depth *h*, opening δ, length *S* and orientation with the reference to the general thermal flow.

The temperature differential on the sample's surface $\Delta T(\tau)$ has been chosen as the criterion of a defect presence.

The sample's surface is heated to the temperature T^* by the thermal flow Q from a heat source. The temperature measurement is made in τ seconds after switching off the heat source. Thus, the mathematical model of the temperature differential $\Delta T(\tau)$

dependence on the vector of defect parameters $\Theta = (h, \delta, s)^T$ (T – transposing sign) in a registered point *A* of the plate's surface can be presented as follows:

$$\Delta T = \sum_{i=0}^{k} b_i f_{iu}(\Theta) + \varepsilon, \quad (6)$$

where ε – is random disturbance.

The random disturbance is assumed to come up to classic regressive analysis standards (normal distribution, noncorrelatedness and variances equality). The value of $\Delta T(\tau)$ for different defect parameters can be obtained experimentally or as a result of numerical solving of the equations (1) – (5) (for instance, it is possible to apply the final elements method).

Using the least-squares method, we have found the estimation β of the vector *b*:

$$\beta = (\Lambda^T \Lambda)^{-1} \Lambda^T \Delta T, \quad (7)$$

where $\Delta T = (\Delta T_1, ..., \Delta T_N)^T$, $\beta = (\beta_0, ..., \beta_k)^T$, $\Lambda - [N \times (k+1)]$-dimensional matrix.

Further, changing testing parameters such as power of the heat source, velocity of its movement, delay τ etc., we have constructed *P* models of the following form:

$$\Delta T^j = f^T(\Theta)\beta^j, \quad j = \overline{1, p}, \quad (8)$$

The covariance matrix of the estimations β^j is defined as:

$$V^j(\beta) = (\Lambda^{jT} \Lambda^j)^{-1} \sigma^2 = C^j \sigma^2, \quad (9)$$

where C^j – is variance matrix ($j = \overline{1, p}$); σ^2 – is variance of the measurement type.

Having the previously obtained value of the vectors β^j ($j = \overline{1, p}$) coordinates, we can use the active thermal technique for estimation of defect parameters. For this purpose let's take the same sample but with unknown defect parameters, and measure ΔT under the same experiment conditions as the models (8) have been constructed. As a result, we need to estimate a vector of parameters Θ of the process:

$$\Delta T = Bf(\Theta) + \varepsilon, \quad (10)$$

where $\Delta T = (\Delta T_1, ..., \Delta T_p)^T$; $B - [p \times (k+1)]$-dimensional matrix.

The model (10) is nonlinear in vector coordinates of the parameters Θ. So, it is expedient to use the iteration procedure to get estimations of the vector $\hat{\Theta}$. The proposed technique is based on the H_∞ estimation theory. It is motivated by the fact that is required the input-output measurements to be bounded only. No other apriori information is needed.

The H_∞ optimization scheme leads to improved performance of the deconvolution process over the existing least squares or H_∞ based solution, thus leading to enhanced defect impulse response thereby improving defect identification [2].

First the H_∞ defect identification problem is formulated as an input-output parameter estimation problem, put into an equivalent state space representation. An H_∞ state space estimation algorithm is then used to solve the deconvolution problem [3].

The estimator should be chosen for the worst possible ε (representing noise and modeling errors). It is required to obtain an estimator, which produces the best estimate $\hat{\Theta}$ in the following sense.

Search for $\hat{\Theta}$ so that:

$$\sup_{\varepsilon\in l_2[0...N]} \frac{\left\|\Theta-\hat{\Theta}\right\|_2^2}{\left\|\varepsilon\right\|_2^2+\left\|\Theta-\mu\right\|_{R_0^{-1}}^2} < \gamma^2 , \qquad (11)$$

for the value of γ that is as close as possible to the minimum possible value γ_0.

The differential game problem that corresponds to (11) is defined in terms of the following cost function [3]:

$$J = \left\|\Theta-\hat{\Theta}\right\|_2^2 - [\left\|\varepsilon\right\|_2^2 + \left\|\Theta-\mu\right\|_{R_0^{-1}}^2] , \qquad (12)$$

The above differential game problem (11) is equivalent to H_∞ optimization [4].

APPLICATION

The above-mentioned technique was applied to identify defect parameters of a sample with the following thermal characteristics:

λ_1=45,4 $W/(m\cdot K)$, ρ_1=7800 kg/m^2, c_1=0,46$\cdot10^{-3}$ $J/(kg\cdot K)$, α=15 $W/(m^2\cdot K)$.

The defect thermal characteristics were:

λ_2=0,925 $W/(m\cdot K)$, ρ_2=1,293 kg/m^2, c_2=1010 $J/(kg\cdot K)$.

At modeling the process (6), the defect parameters ranged:

δ – from 1 mm to 3 mm;

h – from 1 mm to 6 mm;

S – from 2 mm to 8 mm.

The models (8) were defined as follows:

$$\Delta T = \beta_1 h + \beta_2 S + \beta_3 h^2 + \beta_4 S^2 + \beta_5 hS + \beta_6 h^3 + \beta_7 S^3 + \beta_8 hS\delta + \beta_9 h^2 S , \qquad (13)$$

The optimal estimate $\hat{\Theta}$ was found with the use of the following iteration procedure:

$$\hat{\Theta}_{k+1} = \hat{\Theta}_k + R_k^{-1} B(\Delta T - Bf(\hat{\Theta})), \tag{14}$$

$$R_k = M_k - \frac{M_k BB^T M_k}{B^T M_k B + 1},$$

$$M_k = P_k^{-1},$$

$$P_{k+1} = P_k + BB^T - \gamma^2 I,$$

where I – is unit matrix.

Figure 2 shows how definition error of defect location depth h depends on h_r and δ_r. Where h_r, s_r, δ_r - real defect parameters and h_c, s_c, δ_c - parameters obtained by the proposed technique. With a small defect location depth (h_r =1), the error of its definition increases when a value of defect opening comes down.

The dependence of definition error of defect length s on its location depth h is shown on Figure 3. With a small defect location depth, the definition error of defect length increases when a value of defect opening decreases.

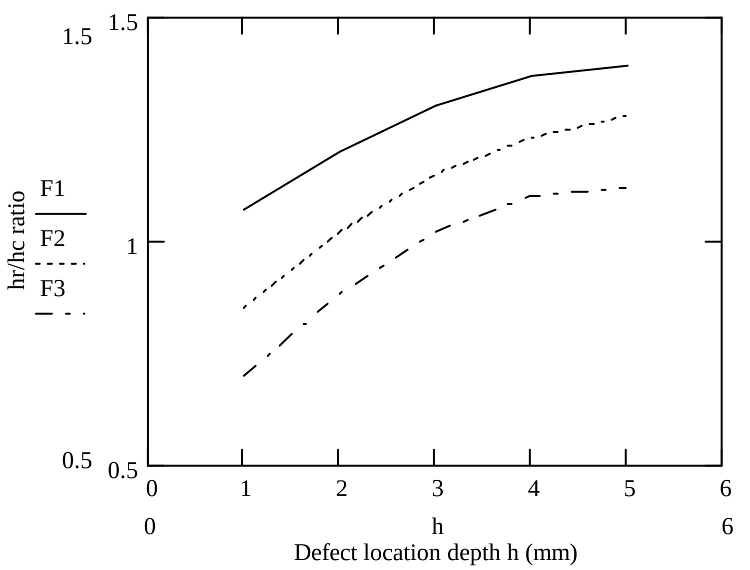

Figure 2. A plot of the h_r / h_c ratio as a function of a defect location depth h.
F1 – δ_r = 3 mm, F2 – δ_r = 2 mm, F3 – δ_r = 1 mm.

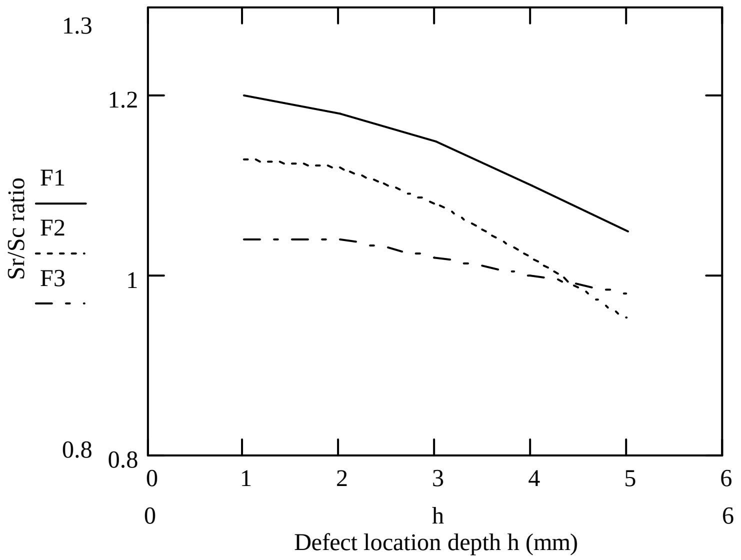

Figure 3. A plot of the s_r / s_c ratio as a function of a defect location depth h.
F1 – δ_r = 1 mm, F2 – δ_r = 2 mm, F3 – δ_r = 3 mm.

CONCLUSION

In this work a new method for estimation of defect parameters has been presented. This technique can be used in active thermal nondestructive testing devices, which detect defects and operate in real time.

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