defectiveness and the $K = (\sigma_{II}^{temp})'/(\sigma_m)'$ ratio, where $(\sigma_m)'$ is the load at which $(\sigma_{II}^{temp})'$ is reached, is assumed to be the adjusted index of defectiveness, then the $\sigma_f - K$ relationship quantitatively reflects the relationship of the structural defectiveness to the strength of a brittle material (Fig. 4). Attention should be devoted to the fact that the data for different samples (ZrC, ZrC + C, NbC, and NbC + C) lies on a common curve.

Conclusions. In sintered refractory carbides in force loading temporary microstresses, the amount of which is sufficient for x-ray diffraction detection based on widening of the line, occur.

The difference in the degree of structural defectiveness is revealed in the difference in the values of σ_{II}^{temp}, which may be of practical interest in investigation of the changes in structure under the influence of heat treatment or thermomechanical working of carbides. For this it is sufficient to compare the values of σ_{II}^{temp} of the same sample subjected to different forms of treatment in loading with the same uniaxial load.

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POROSITY DETERMINATION OF THIN FIBROUS-MATERIAL SHEETS ACCORDING TO THE OPTICAL TRANSMISSION COEFFICIENT

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The rising interest in the use of fibrous materials for filtration of liquids and gases, capillary transport, transpiration, noise absorption, and thermal insulation, etc. is dependent on their capability to completely satisfy the body of requirements for materials of such designation [1]. Since the properties of porous fibrous materials are always associated with structural parameters (pore size, porosity, geometric size and shape of fibers, distribution of fibers in the material, etc.), each specific utilization case, especially for operation under extreme conditions, requires optimization of these characteristics. There is therefore a practical interest in effectively controlling the basic structural characteristics of fibrous materials in the process of formation as well as in a finished piece.

The most common of fibrous-material intermediate product is high-porosity sheet felt (Fig. 1). With one or several layers of such felt, pieces of prescribed structural parameters and geometric shape are produced

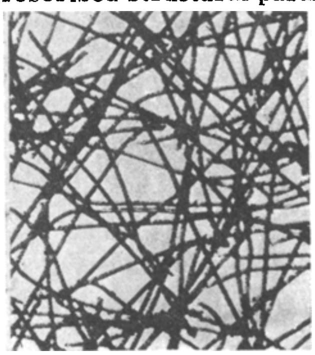

Fig. 1. Structure of a thin sheet of a porous fibrous material.

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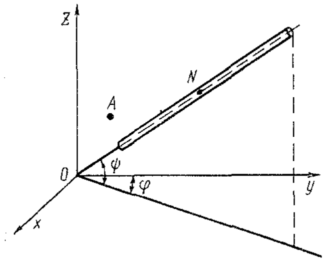

Fig. 2. Derivation of expression (5).

by rolling or pressing and subsequent strengthening treatment, for example, by sintering [2]. Consequently, the fibrous-material properties are formed already during the stage of producing the intermediate product, which is thin-sheeted felt.

Control of the felt structural parameters, and also of pieces made from it in the form of thin sheet, is difficult by traditional methods for known reasons. In addition, possibilities are opened for study of the structure of given objects using their optical properties. In this connection, it is expedient to use the optical methods of laser diffractometry and coherent optics [3, 4], whose essence consists in analysis of the diffraction pattern of a studied object.

The goal of the present work is the structural analysis of porous fibrous materials by mathematical statistics and the theory of random processes. The goal is also establishment of a link between one of basic structural parameters, the porosity, and the optical transmission coefficient. All of this is in the framework of a unique probability-statistical approach.

We introduce a fibrous-material statistical model based on a composite random process, described by the function P(A) [5], which receives a value of zero if a point A is located in a fiber and a value of unity outside of it. During this, we will consider as primary the distribution process of geometric fiber centers in the material bulk, represented as a three-dimensional recurrent flux [6], i.e., a three-dimensional flux with limited aftereffect. By aftereffect we mean here the impossibility of moving the favored close grouping of two and more centers, i.e., mutual fiber penetration. If we even assume that mutual fiber penetration at contact points is still possible, the primary flux will nevertheless have properties of stability, uniqueness, and absence of aftereffect. This enables us to assign the flux to the category of steady-state Poissonian flux, and to describe the probability $P_n(V)$ of n centers falling in each fixed volume V by Poisson's law [7]:

$$P_n(V) = \frac{(\lambda V)^n}{n!} e^{-\lambda V}, \tag{1}$$

where λ is the flux density (average number of centers per unit volume).

The proposed fibrous-material model enables determination of structural parameters such as the statistical process characteristics P(A). Thus, the porosity of a material Θ is the mathematical expectation M[P(A)] of a process P(A), calculated equal to the probability that an arbitrary point A is not located within a fiber. If the primary process is given by a Poissonian flux with the distribution law (1), the mathematical expectation is determined as [8]

$$M[P(A)] = \exp\left[-\lambda \int_0^{2\pi}\int_0^{2\pi} w(\varphi, \psi) \int_V F(\varphi, \psi, V)\, dV d\varphi d\psi\right]. \tag{2}$$

In this equation $w(\varphi, \psi)$ is the common probability density for distribution of the fiber-axis orientation angles φ, ψ (Fig. 2); $F(\varphi, \psi, V)$ is a unity-zero function equal to unity if a fiber with a center at point N captures the point A, and zero in the inverse case. The integral $\int_V F(\varphi, \psi, V)dV$ expresses the volume of a body formed by the geometric locus of fiber centers with orientation angles φ, ψ, encompassing point A. To determine the mathematical expectation of expression (2), we introduce the following conditions: the fibers are cylindrical in shape with diameter d and length l, the location and orientation of the fibers are independent of each other; the fiber axes are parallel to the plane of the sheet and oriented according to the law of equal probability.

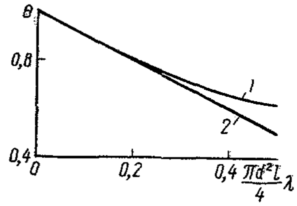

Fig. 3

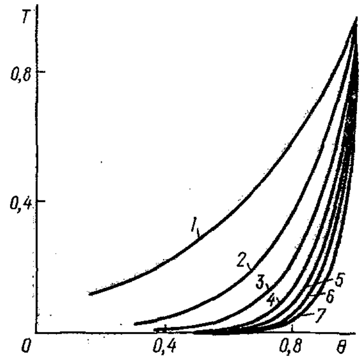

Fig. 4

Fig. 3. Dependence of porosity on density of centers, fiber length, and fiber diameter: 1) model; 2) actual material (d = 0.025, l = 2.5 mm).

Fig. 4. Dependence of the optical transmission coefficient on the porosity of fibrous materials (d = 0.025 mm, l = 2.5 mm): 1) h = 50; 2) 100; 3) 150; 4) 200; 5) 250; 6) 300; and 7) 350 μm.

It follows from these conditions that the common probability density of angles φ and ψ equals

$$w(\varphi, \psi) = \frac{1}{4\pi^2}\delta(\psi), \tag{3}$$

where $\delta(\psi)$ is the delta function of Dirac [4], and

$$\int_V F(\varphi, \psi, V)\,dV = \pi d^2 l/4. \tag{4}$$

Substituting Eqs. (3) and (4) into (2), we obtain

$$\Theta = M[P(A)] = \exp[-\lambda\pi d^2 l/4]. \tag{5}$$

On the other hand, the porosity of a fibrous material, also depending on the density of centers and the geometric sizes of fibers, may be represented by the equation

$$\Theta = 1 - \lambda\pi d^2 l/4, \tag{6}$$

since $\lambda\pi d^2 l/4$ is the portion of unit material volume occupied by fibers, that is, the relative density of the material.

The dependences of Θ on $\lambda\pi d^2 l/4$ from expressions (5) and (6) are shown graphically in Fig. 3. Their comparison enables setting the limits for application of the proposed model.

The optical transmission coefficient, defined as the ratio of that portion of light flux which passes through a permeable object to the total flux, contains information on the integral characteristic of projection of the object's structure on a surface perpendicular to the direction of the light flux. We used this fact to determine the porosity of thin sheets of fibrous materials according to their transmission coefficient. The above statistical model of a porous material enables us to establish a functional interdependence between the porosity and the optical transmission coefficient using a unique probability-statistical approach.

We determine the optical transmission coefficient of a fibrous sheet material T as the mathematical expectation of a process T(A), formed by projections of the initial composite process on a plane parallel to the plane of the sheet and perpendicular to the direction of the light flux.

Thus, the transmission function is also a compuund process in which the first term denotes the two-dimensional Poissonian flow with the density $\lambda \cdot h$ (h is the thickness of a material layer). Since the fiber projections have the shape of rectangles of length l and width d, the average transmission coefficient equals [8]

$$T = M[T(A)] = \exp[-\lambda h l d]. \tag{7}$$

From this it follows that the transmission coefficient T of a fibrous material is determined by the sample thickness h and the density λ of fiber centers with their geometric dimensions l and d. Deriving from (6) the density λ as a function of Θ and substituting in equation (7), we obtain the dependence linking the transmission coefficient T with the porosity Θ:

TABLE 1. Determination of the Porosity of Thin Sheets of Fibrous Materials According to the Optical Transmission Coefficient

Thickness (mm)	Density of centers (mm^{-3})	Transmission coeff.		Porosity	
		calc.	expt.	calc.	expt.
0,250	59,4	0,385	0,383	0,912	0,925
0,250	63,3	0,362	0,371	0,913	0,922
0,270	149,2	0,075	0,078	0,794	0,814
0,280	151,3	0,066	0,065	0,794	0,809
0,260	181,6	0,048	0,051	0,753	0,775
0,280	181,2	0,038	0,043	0,753	0,779
0,230	286,8	0,014	0,012	0,603	0,621
0,250	317,6	0,006	0,007	0,568	0,610
0,260	319,1	0,005	0,007	0,566	0,625

$$T = \exp\left[-\frac{4h}{\pi d}(1-\Theta)\right]. \tag{8}$$

The calculated T = f(h, Θ) dependences are shown in Fig. 4. The presence of a well-defined functional dependence confirms the possibility of determining the porosity by measurement of the transmission coefficient. Writing the porosity as a function of the transmission coefficient, we obtain

$$\Theta = 1 + \frac{\pi d}{4h} \cdot \ln T. \tag{9}$$

In computing Θ by expression (9), we need to know, besides the transmission coefficient T, the sample thickness h and the diameter d of the fibers. The length l of the fibers does not enter the given expression.

To check the resulting analytical dependences, we measured the transmission coefficients for samples sintered from stainless steel fibers 0.025 mm in diameter and 2.5 mm in length. For this, we used an FM-8 spectrophotometer. The illumination source was an incandescent lamp. We computed the sample porosity according to measured data with the aid of expression (9).

Besides this, we determined the average fiber center density in samples for which we had computed the transmission coefficient and porosity using expressions (7) and (6).

Analysis of the results (Table 1) shows that the experimental data satisfactorily agree with the theoretical predictions.

Conclusions. A statistical model of a porous fibrous material is presented which describes, within 6% error at $\theta \geq 0.7$, the porosity θ and the optical transmission coefficient T of a sheet fibrous material as a function of the geometric fiber sizes: i.e., diameter d, length l, sheet thickness h, and average density λ of the geometric fiber centers. A functional dependence f = (h, T) was established which allowed the porosity of thin fibrous material sheets to be determined by measurement of the optical transmission coefficient.

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