The problems of designing coherent spectrum analyzers

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ABSTRACT

The purpose of this article is to improve methods of calculating generalized characteristics of the coherent spectrum analyzers, which define the device's properties and operation. These are the working range of spatial frequencies, the spatial spectral resolution and the energy resolution. Due to these methods, it is possible to choose optimal dimensions and parameters of components of the device to improve the properties of the last.

Keywords: coherent spectrum analyzer, space bandwidth product, spatial spectral resolution, and energy resolution.

1. INTRODUCTION

In recent years, an interest to optical information systems that use light to process information has been increased. There is a rapid development of laser measurement technology in such fields as metrology, microbiology, radiolocation and others[1,2]. Nowadays, the devices for measuring dimensions of fine objects, investigation of microdefects on surfaces, investigation of geometric parameters of spatial quasi-periodic structures that are widely used in vacuum tube for controlling electron beams, etc., got an essential interest. Most of these devices are based on coherent spectrum analyzers.

Coherent spectrum analyzers differ from other data processing systems by their simplicity. When choosing a system for solving one or another problem, it is very important to ensure all necessary system parameters characterizing the effectiveness of the device. Today, the fundamental physical principles of the coherent analyzers that are described in many monographs and articles have been already explored[3,5-8].Nevertheless, not all methods for designing optical systems of the coherent analyzers were developed and described.

2. GENERAL SCHEMES OF CONSTRUCTING THE COHERENT SPECTRUM ANALYZER

The coherent spectrum analyzer consists of a coherent light source (a laser beam), a transparency characterizing by amplitude transmittance, a lens, and a CCD image sensor. All components are consistently located to each other. The lens used for spectral analysis is called the Fourier-lens.

The principle of operation of the coherent analyzer is spectral decomposition of a spatial signal. It enables to analyze at the same time amplitude and phase spectrums of both one-dimensional and two-dimensional signals. The light propagates from the source and diffracts on the transparency with a certain transmittance. The Fourier-lens forms the spatial spectrum of the input signal. After that, the photodetector records the illumination distribution and a computer performs the analysis of spectrum parameters. The signal at output of the optical system coincides (with a constant factor) with the Fourier transform of the input signal. Therefore, the output plane of such system is called the Fourier-plane.

There are two generalized schemes of constructing the coherent spectrum analyzer: a) when the transparency is located in front of the lens; b) when the transparency is located behind the lens.

Figure 1 shows the propagation of coherent radiation in general schemes.
The main component of the optical system is a spherical lens. The lens not only creates an image, but also performs the phase transformation and the Fourier transform.

Let the point light source that located in the plane x_0y_0 has the wavelength λ and the amplitude V_0. The plane x_3y_3 is a spatial range of the field distribution in the plane x_1y_1 for the diagram which is shown in Figure 1, *a*, in the plane x_2y_2 for the diagram which is shown in Figure 1, *b*. The field distribution in the plane of observation x_3y_3 includes quadratic phase distortions of the field that introduce additional errors during the amplitude-phase registration. As a result, this

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complicates reading of measurement data. In the textbook[3] based on the classic Huygens-Fresnel principle there were obtained principal mathematical dependences that describe the propagation of coherent waves in the optical system of the coherent spectrum analyzer taking into account the quadratic phase distortions. This allows to choosethe optimal geometric dimensions of the optical system which eliminate or decrease the magnitude of the quadratic phase distortions of the frequency spectrum.

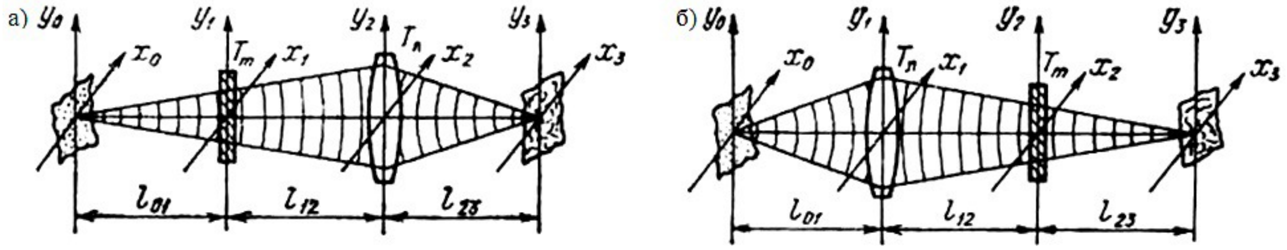

Figure 1.Schematic diagrams of constructing the coherent spectrum analyzer: *a* is when the transparency locates in front of the lens; *b* is when the transparency locates behind the lens

For the diagram when the transparency is located in front of the lens (Figure 1, *a*) the phase factor is eliminated by:

$$\frac{1}{l_{02}}+\frac{1}{l_{23}}=;\frac{1}{f} \tag{1}$$

where$l_{02}=l_{01}+l_{12}$ is a distance from the light source to the plane of Fourier-lens; f is a focal length of the lens.
In this case, the spatial frequencies of spectrum are given by[3]:

$$\nu_x=\frac{x_3}{\lambda l_{12}l_{23}\left(\frac{1}{l_{12}}+\frac{1}{l_{23}}-\frac{1}{f}\right)};\ \nu_y=\frac{y_3}{\lambda l_{12}l_{23}\left(\frac{1}{l_{12}}+\frac{1}{l_{23}}-\frac{1}{f}\right)}. \tag{2}$$

For the diagram in case Figure 1, *b* the phase factor is eliminated by:

$$\frac{1}{l_{01}}+\frac{1}{l_{13}}=\frac{1}{f}, \tag{3}$$

where $l_{13}=l_{12}+l_{23}\,l_{13}=l_{12}+l_{23}$is a distance from the plane of Fourier-lens to the planex_3y_3.
The spatial frequencies of spectrum in case Figure 1, *b* equal:

$$\nu_x=\frac{x_3}{\lambda l_{23}};\ \nu_y=\frac{y_3}{\lambda l_{23}}. \tag{4}$$

By varying the distance between the planes, one can change the scale of spatial frequencies in the plane of observation x_3y_3. The general formula for the scale factor of spatial frequenciesν_x and ν_y in the explored frequency spectrum has a form:

$$\mu=\frac{\nu_x}{x_3}=\frac{\nu_y}{y_3}. \tag{5}$$

The most common scheme of constructing the coherent spectrum analyzer is the scheme in which the transparency is located in front of the Fourier-lens. In such a system, the quadratic phase distortions of the frequency spectrum are removed (when the transparency located in the front focal plane of the Fourier-lens), and by the adjustment of the optical system one can achieve the minimal distortions of its spatial and energy parameters[3].

3. DEFINITION OF GENERALIZED CHARACTERISTICS OF THE COHERENT SPECTRUM ANALYZER

The efficiency of the coherent spectrum analyzer for the solution of one or another problem can be estimated by using the basic characteristics. The basic characteristics of the coherent spectrum analyzers that define their properties and functionalities are the working range of spatial frequencies, space bandwidth, the spatial spectral resolution and the energy resolution.

The spatial working range of spatial frequencies is the range of spatial frequencies within which the optical system of the coherent spectrum analyzer passes through all frequency components of investigated spectrum. The range of spatial frequencies that passed the system will depend on the diameter of the entrance pupil of the lens. At a certain spatial frequency $\nu_{x,max}$ the investigated spectrum will disappear. This maximum spatial frequency determines the working range of spatial frequencies.

Spatial bandwidth production SBD is defined by the number of resolved read-outs that can be formed by the spectrum analyzer[9]. This bandwidth is an analogue of the spatial working range of spatial frequencies. The maximal spatial frequency passed through a COS is limited by a maximal angle of a diffraction grating located in a plane of the input transparency. Let consider a case when a plane wave front goes normally to a diffraction grating which is located in a front focal plane of a lens. We can compose the expression for the first-order diffraction using principal equation of the grating: $d\sin\varphi_{d1} = \lambda$, where $d = 1/\nu_{res}$ – the spatial period of the diffraction grating. Then

$$\nu_{res} = \frac{1}{d} = \frac{1}{\lambda}\sin\theta_{max} \tag{6}$$

The spatial bandwidth of the spectrum analyzer is equal to:

$$SBP = \frac{D_o}{d} = D_o \nu_{res} = \frac{D_o}{\lambda}\sin\varphi_{d1}, \tag{7}$$

where D_o – the diameter of the input діаметр вхідного transparency.

The range of frequencies at which the system can separate two spectral components of equal amplitude with the frequencies ν_x and $\nu_x + \Delta\nu_x$ characterizes the *spatial spectral resolution* $\Delta\nu_x$. Since the radiation detector registers the output signal, the size of the sensitive area (pixel) determines the spatial spectral resolution.

The energy resolution determines the ability of the radiation detector to detect the signals of objects on the background noise. The main criterion that determines the energy solution is the threshold sensitivity of the radiation detector. The threshold sensitivity is called the least radiant flux or the smallest value of the irradiance that can be registered by the receiving system. The minimum signal level corresponding to the threshold sensitivity is the signal level at which the output signal-to-noise ratio equals to one.

4. METHODS OF CALCULATING GENERALIZED CHARACTERISTICSOF THE COHERENT SPECTRUM ANALYZER

To design the coherent spectrum analyzer it is important to provide the required characteristics listed above. Let consider the methods of calculating generalized characteristics for the common scheme when the transparency is located in the front focal plane of the lens (Figure 2). As the transparency, we take a screen with a rectangular aperture with size$a \times b$ and transmittance$t_o(x_1, y_1)$.

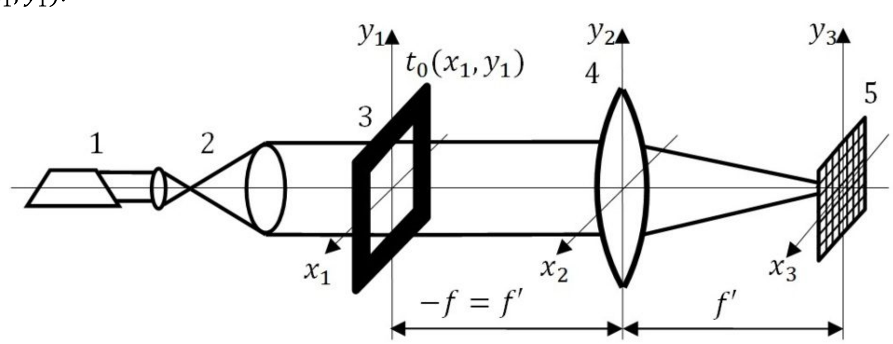

Figure 2. The diagram of the coherent spectrum analyzer: 1 is a laser; 2 is a collimating system; 3 is a rectangular transparency; 4 is a Fourier-lens; 5 is a CCD image sensor

The monochromatic plane wave falls on the transparency, and, as the result, on the transparency one can observe the diffraction. Therefore, the field distribution in the plane of the radiation detector is defined by the expression[3]:

$$V(x_3, y_3) = \frac{1}{j\lambda f'} \iint_{-\infty}^{\infty} V(x_1, y_1) \exp\left[-j \frac{2\pi}{\lambda f'} (x_3 x_1 + y_3 y_1)\right] dx_1 dy_1, \quad (8)$$

where λ is the wavelength; f' is the focal length of the Fourier-lens; $V(x_1, y_1)$ is the field distribution in the plane of the transparency; x_1, y_1 and x_3, y_3 are spatial coordinates in the planes $x_1 y_1$ and $x_3 y_3$ respectively.

The spatial range of the function $V(x_1, y_1)$ is determined by the two-dimensional Fourier transform:

$$F\{V(x_1, y_1)\} = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} V(x_1, y_1) \exp[-j2\pi(\nu_x x_1 + \nu_y y_1)] dx_1 dy_1, \quad (9)$$

where ν_x, ν_y are spatial frequencies.

Comparing the expressions (8) and (9), one can note that spatial frequencies equal:

$$\nu_x = \frac{x_3}{\lambda f'}; \nu_y = \frac{y_3}{\lambda f'}. \quad (10)$$

The intensity of the field distribution in the plane of observation equals to the square of the modulus of the Fourier transform, and the expression of this intensity is as follows: $I(x_3, y_3) = |V(x_3, y_3)|^2$.

Let consider the one-dimensional field distribution along the *x* axis. To find the working range of spatial frequencies, use the laws of the geometrical optics. The optical system of the coherent spectrum analyzer transmits the spatial frequencies only in the range from 0 to $\nu_{x,max}$ .Let consider such propagation of the beam through the Fourier-lens in which the maximum spatial frequency passes the system (Figure 3).

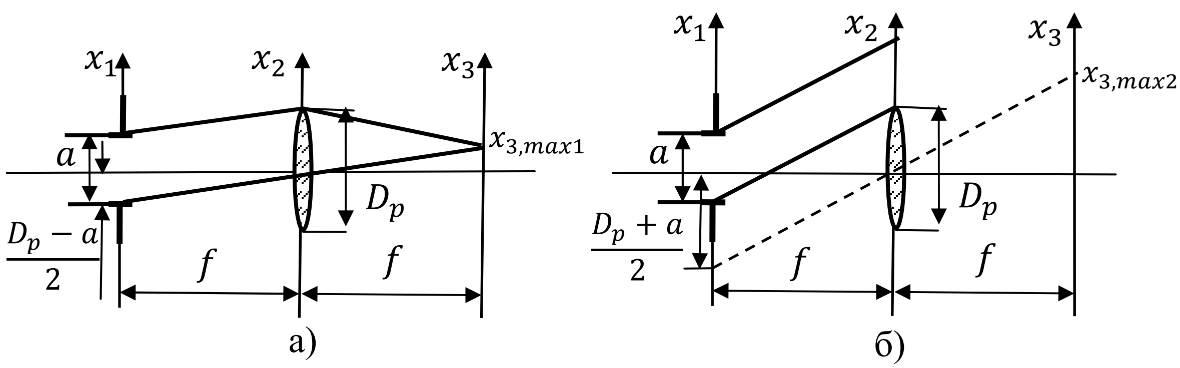

Figure 3. Propagation of the beam through the Fourier-lens: *a* is without vignetting; *b* is with non-zero vignetting

Figure 3 shows that the limiting spatial frequencies will be equal:

$$\nu_{x,max1} = \frac{x_{3,max1}}{\lambda f} = \frac{D_p - a}{2\lambda f}; \qquad \nu_{x,max2} = \frac{x_{3,max2}}{\lambda f} = \frac{D_p + a}{2\lambda f}, \quad (11)$$

where D_p is a diameter of the entrance pupil of the Fourier-lens.

It is necessary to note that no vignetting occurs in the spectral range from 0 to $\nu_{x,max1}$, and non-zero vignetting occurs when the range is from $\nu_{x,max1}$ to $\nu_{x,max2}$. The signal is missed when the frequency exceeds $\nu_{x,max2}$.

The space bandwidth can be found using the formula (7), in which we can calculate the diffraction angle using Fig.3,a.

$$\text{tg}\varphi_{d1} = \frac{D_p - a}{2f} \Rightarrow \varphi_{d1} = \text{arctg}\left(\frac{D_p - a}{2f}\right). \quad (12)$$

If we insert the formula (12) in to the formula (7) we get the following:

$$SBP = \frac{a}{\lambda} \sin\left[\text{arctg}\left(\frac{D_p - a}{2f}\right)\right]. \quad (13)$$

Let find the minimum spatial spectrum of the input signal that is recognized by one pixel. The pixel of the matrix detector has the size $V_D \times W_D$. Taking into account the spatial frequency, which is defined by the expression (10) and the pixel's size, find the spatial spectral resolution:

$$\Delta\nu_x = \nu_{x2} - \nu_{x1} = \frac{x_3}{\lambda f'} - \frac{x_3 - V_D}{\lambda f'} = \frac{V_D}{\lambda f'}, \tag{14}$$

where $\nu_{x1}, \nu_{x2}$are spatial frequencies corresponding to adjacent pixels.

The purpose of energy calculations is to determine the magnitude of an electrical signal and the signal-to-noise ratio at output of the CCD image sensor depending on the spatial frequency of the input optical signal.

This requires to choose an etalon optical signal with known spatial spectrum. The signal can be a slit, a rectangular aperture, a one-dimensional sinusoidal functions of the transparency's amplitude transmittance, etc.

As the etalon test object let take the rectangular aperture with the size $a \times b$, which has spatial spectrum described by sinc functions.

The drawback of this signal is that the amplitude of the light field at a certain point of analysis depends on the aperture's size$a \times b$, thatdoes not allow to extend the results of energy calculations for arbitrary input signals.

Let define the signalu_s and the signal-to-noise ratio *SNR* at the Nyquist frequency, which is $\nu_N = 1/2a$. Such an approach is extensively used in determination the spatial resolution of television end thermovision digital cameras[10]. The Nyquist frequency in the plane of analysis $x_3 y_3$ has coordinates:

$$x_{3N} = \frac{\lambda f}{2a}. \tag{15}$$

Let determine the magnitude of spatial spectrum of the rectangular aperture at the Nyquist frequency. The amplitude transmittance of such aperture is given by:

$$t_o(x_1, y_1) = \begin{cases} 1, \text{if} |x_1| \le \frac{a}{2}, |y_1| \le b/2; \\ 0, \text{in other cases.} \end{cases} \tag{16}$$

The spatial spectrum of function (16) is found by the two-dimensional Fourier transform[10]:

$$t_o(\tilde{\nu}_x, \nu_y) = F\{t_o(x_1, y_1)\} = ab \frac{\sin(\pi a \nu_x)}{\pi a \nu_x} \frac{\sin(\pi b \nu_y)}{\pi b \nu_y} = ab \operatorname{sinc}(a\nu_x) \operatorname{sinc}(b\nu_y). \tag{17}$$

where $\operatorname{sinc}(z) = \frac{\sin(\pi z)}{\pi z}$ is the sinc-function.

In the point (0, 0) we have the spatial spectrum$\tilde{t}_o(0,0) = ab$. The normalizing spectrum is defined as:

$$\tilde{t}_{on}(\nu_x, \nu_y) = \frac{\tilde{t}_o(\nu_x, \nu_y)}{\tilde{t}_o(0,0)} = \operatorname{sinc}(a\nu_x) \operatorname{sinc}(b\nu_y). \tag{18}$$

where $\nu_x = \frac{x_3}{\lambda f}$is the spatial frequency.

The normalizing one-dimensional spectrum at the Nyquist frequency has the magnitude:

$$t_{on}(\tilde{\nu}_N, 0) = \operatorname{sinc}(a\nu_N) = \frac{\sin(\pi a \frac{1}{2a})}{\pi a \frac{1}{2a}} = \frac{2}{\pi}. \tag{19}$$

This means that at the Nyquist frequency the spectrum of the signal reduces in $\pi/2$ times against the maximum magnitude.

Let consider a sequence of the radiation's transformation from the laser to the plane of analysis (Figure 2).

If the illumination system forms the monochromatic plane wave with the amplitude V_0 that normally illuminates the transparency, then the field amplitude behind the transparency equals:

$$V(x_1, y_1) = V_0 t_o(x_1, y_1), \tag{20}$$

where the amplitude transmittance of the transparency is described by the function (13).

Taking into account the expression (18), one can find the field amplitude in the plane of analysis:

$$V(x_3, y_3) = \frac{V_0}{\lambda f} F\{t_o(x_1, y_1)\} = \frac{V_0 ab}{\lambda f} \operatorname{sinc}(a\nu_x)\operatorname{sinc}(b\nu_y). \tag{21}$$

By using the transmittance of the lens τ_o, the irradiance in the plane x_3y_3, where the CCD image sensor is placed, can be written as:

$$E(x_3, y_3) = \tau_o E_0 \left(\frac{ab}{\lambda f}\right)^2 \operatorname{sinc}^2\left(\frac{ax_3}{\lambda f}\right) \operatorname{sinc}^2\left(\frac{by_3}{\lambda f}\right), \tag{22}$$

where $E_0 = |V_0|^2$ is the irradiance of the transparency.

The expression (19) does not take in consideration vignetting of radiation which is practically missed at the small size of the test object, i.e. when$\sqrt{a^2 + b^2} \ll D_p$.

For the analysis of the illumination distribution$E(x_3, y_3)$,let use the CCD image sensor with the following parameters: the spectral The spectral sensitivity $R_D(\lambda)$,R_D V·cm^2/μJ; the root-mean-square noise signalu_n,μV; the integration timeT_i, ms; the pixel size$V_D \times W_D$, μm^2; the dimension of the image zone$X_D \times Y_D$, mm^2.

To find the magnitude of the radiant exposure that provides the irradiance described by the expression (22), one can use:

$$H(x_3, y_3) = E(x_3, y_3)T_i. \tag{23}$$

Then the signal at the output of the pixel equals:

$$u_s = R_D H(x_3, y_3) = R_D E(x_3, y_3)T_i. \tag{24}$$

where R_D is the spectral sensitivity at the wavelength λ.

Finally, the signal-to-noise ratio is given by:

$$SNR = \frac{u_s}{u_n}. \tag{25}$$

5. THE EXAMPLE OF CALCULATIONS

The input data for calculations has the following parameters:

- the parameters of the laser beam: the wavelength λ = 635 nm, output power Φ = 4,5 mW;
- the parameters of the Fourier-lens: the diameter of the entrance pupil D_p= 57,6 mm, the focal length f' = 160 mm, the transmittance at the wavelength 635 nm τ_o= 0,8;
- the size of the transparency $a \times b$= 26×26 mm^2;
- the etalon input signal a rectangular aperture with the size 0,1×0,1 mm^2;
- the parameters of the CCD image sensor: the pixel size $V_D \times W_D$ = 14×14 μm^2; the integration time T_i= 33 ms; the root-mean-square noise signal u_n = 200 μV; the spectral sensitivity at the wavelength 635 nm is R_D= 6 V·cm^2/μJ.

To find the working range of spatial frequencies, let calculate the limiting spatial frequencies by the expressions (11):

$$\nu_{x,max1} = \frac{57{,}6-26}{2\cdot635\cdot10^{-6}\cdot160} = 156 \text{ mm}^{-1}; \quad \nu_{x,max1} = \frac{57{,}6+26}{2\cdot635\cdot10^{-6}\cdot160} = 411\text{mm}^{-1}.$$

Thus, no vignetting occurs in the range from 0 to 156 mm^{-1}, and non-zero vignetting occurs when the range is from 156 mm^{-1} to 411 mm^{-1}. The signal will be missed when the frequency exceeds 411 mm^{-1}.

The spatial bandwidth can be found using the formula (13):

$$SBP = \frac{26}{635\cdot10^{-6}} \sin\left(\operatorname{arctg}\frac{57{,}6-26}{2\cdot160}\right) = 402.$$

To find the minimum spatial spectral resolution, use the expression (14):

$$\Delta\nu_x = \frac{14\cdot10^{-3}}{635\cdot10^{-6}160} = 0{,}138 \text{ мм}^{-1}.$$

The algorithm for calculating the signal-to-noise ratio is as follows:

1. Calculate the irradiance of the transparency by the methods described above:

$$E_0 = \frac{\Phi}{A_{tp}} k_e = \frac{4{,}5 \cdot 10^{-3}}{26 \cdot 26 \cdot 10^{-6}} 0{,}5 = 3{,}33 \ \frac{W}{m^2},$$

   where k_e = 0,5 is the efficiency coefficient providing uniform illumination of the transparency; A_{tp} is the transparency area.

2. Find the irradiance of the CCD image sensor, where the Nyquist frequency located along the coordinate x_3, using the expression (22):

$$E(x_{3N}, 0) = 0{,}8 \cdot 3{,}33 \left(\frac{0{,}1 \cdot 0{,}1}{635 \cdot 10^{-6} \cdot 160}\right)^2 \left(\frac{2}{\pi}\right)^2 = 1{,}05 \cdot 10^{-2} \frac{W}{m^2}.$$

3. Find the radiant exposure by using the expression (23):

$$H = 1{,}05 \cdot 10^{-2} \cdot 33 \cdot 10^{-3} = 3{,}46 \cdot 10^{-4} \frac{J}{m^2}.$$

4. Calculate the magnitude of the signal at the output of the CCD image sensor in accordance with the expression (24):

$$u_s = 600 \cdot 3{,}46 \cdot 10^{-4} = 0{,}21 \, V.$$

5. Determine the signal-to-noise ration by the expression (25):

$$SNR = \frac{0{,}21}{200 \cdot 10^{-6}} = 1050.$$

6. CONCLUSIONS

1. The working range of spatial frequencies is limited by the parameters of the optical system. The larger is the diameter of the aperture diaphragm and the shorter focal length of lens, the greater will be the value of the limiting maximum spatial frequency. At the same time, one should taking into account the size of the matrix detector, which also influences on the required range.
2. Increasing of the spatial spectral resolution, i.e. to reduce Δv_x, is possible by choosing the appropriate radiant detector with the smaller pixel size and by increasing the focal length of the Fourier-lens.
3. The energy resolution depends on the parameters of the radiant detector and the aperture of the Fourier-lens, i.e. the larger is the aperture of the Fourier-lens, the higher will be energy resolution of the device.

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