# *Comments on the Shoucri's interpretation of the contour integral in diffraction theory.
# (On the bicentenary of Th.Young's diffraction paradigm)

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**Abstract**

The Shoucri's interpretation of the contour integral arising within the framework of the Young-Rubinowicz model of diffraction phenomena is considered. Crucial hypothetical experiment intended to prove the Shoucri's interpretation is discussed. It is shown that the initial Young's idea on the nature of diffraction formalized by Rubinowicz possesses undoubted advantages in respect to the Shoucri's interpretation is being undergone the direct experimental verification.

**Key words:** diffraction, Huygens-Frensel principle, edge diffraction wave, holography

## Introduction

The Kirchhoff diffraction theory, being formal expression of the Huygens-Frensel principle, lies in the basis of solving most problems of coherent optics. The Kirchhoff diffraction integral is taken onto the surface covering an aperture at opaque screen. It corresponds to computation of a diffraction field at wave zone behind a screen as the result of coherent superposition of the contributions (wavelets) from secondary sources located within an aperture. To a certain extent arbitrarily, one prescribes some amplitudes, phases, and inclination coefficients to these secondary sources It is well known [I], the Huygens secondary sources, being considered in the optical problem, are fictitious. The Kirchhoff diffraction theory is thought to lack direct physical sense [2,3]. Nevertheless it is a useful receipt for solving of optical problems under paraxial approximation.

Computation of a diffraction field is considerably simplified in some cases if, following Rubinowicz [4], one reduces two-dimensional Kirchhoff integral to one-dimensional (in general, curvilinear) one taken over the rim of a diffraction aperture at opaque screen. This reducing is implemented on the basis of well-known Stokes theorem of the field theory. The Rubinowicz representation of Kirchhoff diffraction integral [2] is the formal expression of the diffraction principle having been put forward by Thomas Young in 1801-1802. According to the Young principle, a diffraction field behind the screen is regarded as the result of interference between geometric-optical wave (GOW) propagating from the primary source within the directly illuminated area, and the edge diffraction wave (EDW) that

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is thought to be re-scattered by a rim of the screen both to the directly illuminated area and into geometrical shadow region.

As it is known [2,5-7], the Huygens-Frensel principle and the Young diffraction principle are alternative ones, being derived from different heuristic ideas on the nature of diffraction. But the corresponding models of diffraction are not mutually exclusive, as each of them may be reduced to another by direct mathematical transformations. The review of studies including theoretical and experimental grounding of the Young model of diffraction (the EDW-model) and discussion of interrelations of this model with alternative ones may be found in [6-14].

The purpose of this paper is to discuss the Shoucri's interpretation of a contour integral to which, following to Rubinowicz, two-dimensional aperture integral is reduced. This interpretation had been put forward for the first time in 1968 [15]. However, due to the absence of common opinion on the physical adequacy of the Young model of diffraction (see, in part, [13] and discussion in [14]), Shoucri has recently reproduced his interpretation in methodically simplified form [16]. The subject of discussion of the Shoucri interpretation of a contour integral in diffraction theory is caused, beside bicentenary jubilee of Th. Young's diffraction paradigm, by the following:

1. The Rubinowicz representation of Kirchhoff diffraction integral is out of objections as asymptotic method for computation of a diffraction field [17]. However, in contradiction with the views of Landsberg [17], the Young-Rubinowicz model of diffraction possesses a deep physical sense [11,18]. High heuristic efficiency of the Young-Rubinowicz model is proved by successful, in our opinion, development of promising optical correlation techniques of data processing on the basis of this model [8,12,19].

2. The Frensel argumentation, seemed to be disaproving the Young's model of diffraction [20], did not stand examination by the time Restricted validity of the main Frensel objection against the Young interpretation of diffraction had been proved experimentally in laser epoch [8,21].

3. Conservative (to say, reactionary) Shoucri's interpretation occurs to be conceptually close to the criticism of the Young model of diffraction from a position of one of the newest branches of optics namely, so-called singular optics [13,22].

## Difficulties of the Young-Rubinowicz model of diffraction

Important disadvantage of the Young-Rubinowicz model of diffraction lies in the fact that amplitudes of both components, in which a diffraction field is decomposed, have discontinuities at the geometrical shadow boundary. So, the GOW's amplitude is equal to the amplitude of the probing wave every where at the directly illuminated area, and the stepwise changes to zero at the geometrical shadow boundary. Much worse, in the typical case of normal incidence of a probing wave at the edge of an opaque screen the EDW complex amplitude undergoes cotangent dependence against a half of a diffraction angle [9] corresponding to the second-kind (infinite) disrupt of this component of a diffraction field. As it is asserted in [13,22], it makes impossible for the discussed field components to exist independently (to propagate) and to be experimentally observed in absolute sense, ι e within a whole angle range [$-\pi$, $+\pi$] including geometrical shadow boundary. Really, amplitude discontinuity of a field or its any component contradicts to the concept of everywhere continuous wave motion. Totally, everywhere continuous diffraction field is obtained only as the result of combination of GOW and EDW, where the discontinuity in any component is compensated by the discontinuity in other one [5,11].

The main remarks on the objections against the Young-Rubinowicz model of diffraction are reduced to the following. At first, discontinuity of a GOW and an EDW at the geometrical shadow boundary is the direct formal consequence of discontinuity of the Kirchhoff boundary conditions [2,3,5]. Thus, from a mathematical point of view, the validity of the Rubinowicz representation is the same as the validity of initial Kirchhoff integral. Because of that, an error arising in practice using the Rubinowicz representation is concentrated within the nearest vicinity of the geometrical shadow boundary whose angular extension is of the order of $10^3$ rad. The validity of the EDW-model beyond this very narrow angular range has been proved both theoretically and experimentally [4-12].

Then, just attraction of the EDW-concept permits to eliminate the contradictions inherent the Kirchhoff diffraction theory [10]. This point being of importance for further consideration, let us discuss it in details. Following Kirchhoff, the field within a diffraction aperture is equal to undisturbed incident field, and the field just behind an opaque screen is equal to zero. Then, both the wave and its first derivative undergo discontinuity at the geometrical shadow boundary. It is well known [2,3,5] that the diffraction problem cannot be formulated consistently as the problem of solving a wave equation under such boundary conditions. It is also known [5] that the field in aperture is really perturbed not only near diffracting edge, but also at considerable (arbitrary) distances from it. The question arises: why the Kirchhoff theory gives the true solutions for numerous diffraction problems in spite of mathematical inconsistency and practical fault of the boundary conditions? Wolf gives the following answer [10]. In fact the existent EDW superimposing with undisturbed GOW is expanded into angular spectrum of plane waves, whose amplitudes are dependent on the diffraction angle. The plane-wave component of the EDW's angular spectrum propagating along an aperture possesses the well-known structure of the surface (evanescent, exponentially decaying) wave [3,5]: its equiphase surfaces are orthogonal to the aperture, while its surfaces of equal amplitudes are parallel to the screen plane. Evanescent wave has a noticeable amplitude only within an aperture and in its nearest vicinity, whose extension is comparable with a wavelength of the probing radiation. But at the wave zone, i.e. at distances behind an aperture exceeding several wavelengths, the amplitude of this EDW's component vanishes. That is why, in spite of inconsistency of the boundary conditions, the Kirchhoff's theory provides accurate solution of the diffraction problem under paraxial approximation.

The mentioned difficulties of diffraction theories associated with the names of Kirchhoff and Rubinowicz had been overcome (at the middle of the 20$^{th}$ century) within the framework of the so-called diffusion model of diffraction. Accordingly to modern views [6,23,24], gradient of a field amplitude at the boundary «light-darkness» is just the actual source of diffraction, while the presence of the edge of an opaque obstacle disturbing homogeneity of a field propagation space is only the pre-condition of such gradient. Within the region of large gradient, the amplitude diffusion through the geometrical shadow boundary (so-called diffusion without mass-transfer) takes place. This interpretation of diffraction phenomena within the nearest vicinity of the geometrical shadow boundary is grounded by neglecting the second-order derivative of a field at wave equation on the space coordinate corresponding to the propagation direction of the incident wave. As a result, a wave (hyperbolic-kind) equation is transformed into diffusion or heat-transfer (parabolic-kind) equation. The Leontovich's parabolic equation and approximate boundary conditions lie in the basis of a diffusion theory of diffraction. The most important problems of radio-physics and laser optics have been solved on the basis of this theory.

## The essence of the Shoucri interpretation

Formalism used by Shoucri for the discussion of physical adequacy of the Rubinowicz's contour integral [15,16] does not exceed the limits of those expounded at any text-book in optics, when the Huygens-Frensel principle is considered. That is why, intuitive qualitative argumentation will be sufficient in our discussion.

Shoucri analyzes a plane where an opaque screen is placed into infinite set of Frensel zones and adds the contributions from all zones at the observation point. The contributions coming from the beginning of *m* zones covered by an aperture are added in a common way, and the contributions coming from the following zones, from the ($m$+ l)-th to infinity, blocked by an opaque screen are subtracted from the total (undisturbed by the screen) field. The key feature of this consideration, explicitly formulated by Shoucri, lies in exclusion *ad hoc* from analysis of «the radiating edge» [5,11] and its effect on a diffraction field. Evidently, the obtained result occurs to be equivalent to the Kirchhoff solution and, of course, to the solution derived by applying the Rubinowicz contour integral. For this reason, Shoucri asserts that the contour integral does not represent physically the existent wave motion, but it rather provides the proper formal description of the resultant effect of «subtracting» the contributions from the Frensel zones blocked by the screen from the total field Literally, Shoucri writes [16] «Since the important assumption is made to neglect any contribution of a scattered wave from the edge of the aperture, then in all evidence this neglected scattered wave cannot magically appear again».

It is interesting, to admit that this interpretation of the contour integral in the diffraction theory is being reproduced by Shoucri during more than thirty (!) years. Nevertheless, neither Shoucri himself nor other researchers did not propose any experiment up-to-now proving the validity of this interpretation. Below we will discuss the following experiment and show that it is only hypothetical («brain»), impracticable experiment. Concerning this problem, the discussed experiment is similar to the famous «twin paradox» in the special relativity theory being formally irreproachable, this experiment does not undergo practical realization.

## Criticism of the Shoucri interpretation

The Shoucri's interpretation would be verified in the following way. Let us assume that undisturbed wave of unitary amplitude from the primary source, hereinafter Wave I, is superimposed co-axially (for example, using the Mach-Zehnder interferometer arrangement) with the copy of a part of this wave, hereinafter Wave II. Let us also assume that Wave II is of the same wave front and amplitude as Wave I, being out-of-phase by $\pi$ with this wave and undergoing discontinuity at the geometrical shadow boundary with the amplitude jump from unity to zero. Naturally, perfect interference compensation of the part of Wave I takes place, the amplitude gradient arises at the boundary «light-darkness», and, in consequence, the «cut» Wave I diffuses in the dark region. Following the Shoucri's interpretation, we may assume that two identical in wave fronts and amplitudes but out-of-phase by $\pi$ integer or non-integer [15,16] sets of the Frensel zones associated with Wave I and Wave II correspond in this hypothetical experiment to the dark region. Agreeing with the Shoucri conclusion, just the additional field (Wave II) associated with the Frensel zones of «extinguished» domain is the source of diffraction. The result of the described experiment would be equivalent to the experiment with blocking of a part of Wave I corresponding to Wave II by an opaque screen with a sharp edge.

It is obvious, the described «brain» experiment cannot be realized in practice. To carry out this experiment, one has, at least, to have the possibility for preparing sharply cut Wave II with the amplitude gradient close to the Dirac's delta-function [25]. However, to make such assumption is the same as to suppose the possibility for the existence of diffraction's propagation of sharply cut wave. Thus, we reveal *vicious circle* of the Shoucri's interpretation. In fact, the last mentioned possibility is quite excluded just by diffraction phenomena. The best practical approximation to the formulated experimental conditions lies in usage of the material opaque screen at the reference arm of an interferometer to obtain the boundary «light-darkness». However, such boundary occurs to be blurred to a certain extent in any physical experiment just due to diffraction. The assumption of the implementability of the discussed experiment is equivalent to the assumption of the possibility of classical (non quantum-mechanical) effect of light-by-light diffraction without interaction of radiation with the matter. Of course, this assumption is quite fantastic. Besides, the inevitability for the usage of material screen (at least, as the aid) obviously contradicts the Shoucri interpretation, for verification of wich the considered experiment is intended. In fact, the inevitability of placing the material screen onto the wave (or its part), which breaks homogeneity of wave propagation space just proves physical existence, physical reality (in contradiction with [13,15,16,22]) of the wave motion component with the center of divergence at the edge of an opaque material screen.

It is quite clear that the Shoucri's interpretation, providing accurate formal description of the diffraction phenomena within the nearest vicinity of the geometrical shadow boundary, does not reveal the actual source of the diffraction, *viz.* the amplitude gradient of a field behind the screen. Besides, the Shoucri's interpretation lacks the definition of an opaque screen that is really the most difficult and unsolved problem of diffraction theory up-to-now [5,9,23]. The opaque screen is equivalent to the superposition of two out-of-phase by $\pi$ waves of equal amplitudes only formally [1].

Further on, opposed to virtual Young's edge retransmitters [12,14,26,27]), Frensel zones associated with the opaque screen and regarded by Shoucri as the energy supplier for realization of diffraction are unobserved because of undergoing imaging and, similarly to Huygens secondary sources, are fictitious radiators.

Insolvency of the Shoucri interpretation may be also shown taking into account a well-known fact that a diffraction field at the geometrical shadow domain depends only on the field magnitude at the diffracting edge rather than on the field distribution within an aperture, in contradiction to the Frensel-Kirchhoff theory of diffraction. This fact had been demonstrated for the case of far-field aperture diffraction of the converging spherical wave (everywhere outside the central maximum of the Fraunhofer pattern) [26,27] and, with special persuing, for the case of knife-edge diffraction of the Gaussian beams [28]. Namely, if a knife edge intersects the intensity maximum of the Gaussian beam, then a diffraction field at the geometrical shadow domain occurs to be the same as in the case when a knife edge is illuminated by plane or spherical wave of the constant amplitude equal to the maximal amplitude of the Gaussian beam [28]. Of course, the Comu's spiral is deformed in the case of the Gaussian beam, and the Shoucri's computation must be modified. However, the independence of a diffraction field within the geometrical shadow region on specified structure of the incident wave outside the nearest vicinity of the diffracting edge is the most valued, crucial argument proving definitely physical inadequacy of the Shoucri's interpretation.

At last, the key of the Shoucri assumption concerning the absence of the edge-diffraction effect must be regarded as one preconceived and insolvent theoretically as well as incorrect practically, as it follows from [5,10] and is discussed in the second section of our paper.

## Conclusions

Shoucri interpretation of the contour integral figuring in the Young-Rubinowicz theory of diffraction is of speculative nature, while its validity cannot be proved by any real physical experiment in principle. The only proof of this interpretation proposed in our paper is based on hypothetical experiment, which does not undergo practical implementation.

Other disadvantages of Shoucri's interpretation, are such as

- unobservability of the Huygens-Frensel (-Shoucri) fictitious sources, *viz.* Frensel zones associated with the field within an aperture unjust behind an opaque screen,
- ignoring the fact that a diffraction field at the geometrical shadow region is independent on the characteristics of the field within the aperture everywhere outside the nearest vicinity of the diffracting edge,
- groundless, preconceived and obviously erroneous ignoring of the edge-diffraction effect, which has been proved both theoretically and experimentally,
- demonstrate physical inadequacy of this interpretation and depreciate the Shoucri's objections of the reality of the wave motion component associated with the EDW

At the same time, correctness and fruitfulness of the Young's model of diffraction as well as the existence of the wave motion component described by the Rubinowicz's contour integral are proved by

- observability of virtual edge retransmitters [2,5] and the possibility of holographic reconstruction of them [8,12,14,19,21,26,27],
- inevitability for attracting the EDW-concept for excluding contradictions (at the stage of formulation of the boundary conditions) inherent the Kirchhoffs diffraction theory [9-11],
- fitness of the principal results of the Young-Rubinowicz theory both to the rigorous Sommerfeld's solution of the problem of diffraction at perfectly conducting half-plane [2,4,5,13] and to the diffusion model of diffraction [6,23-25]

It shows undoubted advantages of the Young-Rubinowicz interpretation of the diffraction phenomena and gives hope for new useful applications of the EDW-model in solving various problems of modern optics two centuries after the initial Thomas Young's idea of the diffraction nature.

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