Reincarnations of optical singularities

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ABSTRACT

In this paper, the chain of mutual transformations of optical singularities is demonstrated, including coherence singularities, polarization singularities of different kinds, and phase singularities. It is shown in what a way one can transform some type of optical singularities into another by changing one of the experimental parameters. Convenient experimental technique for diagnostics of such singularities is also described. It is shown that some of the considered singularities are generic ones, while other are non-generic, implying specific experimental conditions.

Keywords: singular optics, phase singularities, polarization singularities, coherence singularities

1. INTRODUCTION

Optical singularities became one of the most attracting phenomena studied in modern optics[1]. Beside of conventional phase singularities inherent in a common complex amplitude of monochromatic, uniformly polarized ("scalar") and completely spatially coherent optical beams, $a\exp(i\varphi)$, other singularities, such as polarization singularities in non-uniformly polarized fields, coherence singularities in partially spatially coherent fields and white-light vortices are intensively investigated. Generally, phase singularities are inherent in any complex parameter of an optical field[2,3]. It seems clear intuitively that various kinds of optical singularities are interconnected and must transform one into another under some simple experimental conditions. The purpose of this paper is to demonstrate such transformations in combined vortex beams assembled from two weighted coaxial Laguerre-Gaussian modes, LG_0^1 and LG_1^1, for coherent and incoherent mixing and for controlled states of polarization of them. Earlier some properties of such combined beams were investigated experimentally for a few selected situations[4-6]. Here we consider the complete set of transformations of singularities in such beams, including

- common phase singularities, which take place in the case of two mutually coherent modes with the same state of polarization and specific phase relations;
- polarization singularities, which take place in the case of two mutually coherent modes with the orthogonal states of polarization and specific phase difference;
- new type of polarization singularity, *viz.* singularity of the complex degree of polarization defined in Section 2 just for the purpose of this consideration, resulting from incoherent mixing of the partial modes with the orthogonal states of polarization;
- coherence singularity resulting from incoherent mixing of the partial modes with the same states of polarization.

Note that in the third case coherence singularity takes place also, but the physical meaning of the singularities of the complex degree of coherence and of the complex degree of polarization is quite different, as the polarization singularity is the one-point phenomenon while the coherence singularity is two-point one. It will be shown that the first and the second cases (coherent mixing of the partial modes) result in non-generic singularities ("generic means that the object in question is structurally stable to small perturbations and occurs without special preparation or conditions; it is typical and *just happens*"[7]), being sensitive to changing of the parameters of the mixed modes and destroying for such changes. At the same time, the third and the fourth cases result in generic singularities, which are stable against changing the parameters of incoherently mixed modes.

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Eighth International Conference on Correlation Optics, edited by Malgorzata Kujawinska, Oleg V. Angelsky,
Proc. of SPIE Vol. 7008, 70080A, (2008) 0277-786X/08/$18 doi: 10.1117/12.796647

2. COMPLEX DEGREE OF POLARIZATION

The degree of polarization, P, is conventionally defined in terms of the Stokes parameters as the *real, non-negative* value:

$$P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \equiv \sqrt{\sum_{i=1}^{3} s_i^2} \,, \tag{1}$$

where $s_i = S_i / S_0$, $i = 1, 2, 3$ are the second, third and fourth normalized Stokes parameters. P changes from zero for completely unpolarized beam to unity for completely polarized one. In the problem of interest, however, it is expediently to introduce *the complex degree of polarization* (CDP) by the definition:

$$\mathscr{P} = \frac{S_1 + S_2 + S_3}{S_0} \equiv \sum_{i=1}^{3} s_i \,. \tag{2}$$

(Note, the definition (2) is not the same as introduced in Ref. 8 for the complex degree of *mutual* polarization, CDMP, which is two-point characteristics of the beam.) The CDP can be both positive and negative or equal zero taking into account, that the second, third and fourth Stokes parameters can be both positive and negative or equal zero. More specifically, $\mathscr{P}$ changes from –1 to +1. If one specifies some completely polarized beam with the CDP $\mathscr{P} = +1$, then the *orthogonally* completely polarized beam is characterized by the CDP $\mathscr{P} = -1$. As usually, for linearly polarized field $S_3 = 0$ and for circularly polarized field $\sqrt{S_2^2 + S_3^2} = 0$. The beams with $\mathscr{P}$ less than unity by modulus are partially polarized. Such representation is useful for description of partially and non-uniformly polarized optical fields, where both the state of polarization and the degree of polarization change from point to point. For such fields, beside of convenient polarization singularities, such as $C-$points (points with the circular polarization) and $L-$contours (lines along which the field is linearly polarized with gradually changing azimuth of polarization)[7], one can define the new type of polarization singularity, *viz. the singularity of the CDP* or $U-$*singularity*, which takes place when $\mathscr{P} = 0$ and the state of polarization is degenerated (undetermined). The structure and properties of the $U-$singularities, their realization in random light fields will be considered in details in regular paper, now in preparation.

Traveling from point to point in the partially, spatially non-uniformly polarized field can be imaged as the walking on the *deformed* Poincare sphere whose radius is proportional to $\mathscr{P}$ for the each point rather than constant. As conventionally, the completely polarized component of the field is imaged alone at such deformed sphere. In other words, the partially, spatially non-uniformly polarized field at the each point can be represented in the Stokes space, *viz.* at the Poincare sphere by the polarization vector $\mathbf{p} = \{s_1, s_2, s_3\}$ drawn from the center of the Poincare sphere to the point at this sphere imaging the specific state of polarization with the length $\mathscr{P}$. Crossing the point with $\mathscr{P} = 0$ ($U-$singularity), the second, third and fourth Stokes parameters change their signs.

This approach provides complete description of polarization structure of non-uniformly polarized optical fields, in part a random scalar (uniformly polarized) speckle field with the orthogonally polarized reference wave, where the $U-$singularities have a form of closed contours. In this paper we demonstrate for the first time such singularities at the simple modes case when such singularity arises in the combined singular beams assembled from two weighted incoherent Laguerre-Gaussian modes with orthogonal states of polarization.

3. EXPERIMENTAL

Basic arrangement for experimental study of the transformations of optical singularities is shown in Fig. 1. A linearly polarized zero transverse mode beam from a He-Ne laser is split into two beams, where the computer-generated holograms for reconstructing the Laguerre-Gaussian modes LG_0^1 and LG_1^1 are inserted. In our experiment, the power ratio of these modes was 1:0.45. When such modes are incoherently coaxially mixed, then the combined beam is formed with the radial intensity distribution close to such distribution for isolated LG_0^1 mode[4,5], see Fig. 2. Using the Porro prism, one can control the path delay to provide both coherent and incoherent mixing of the partial modes. A half-wave is used for changing the state of polarization of one of two modes. Following to the approach introduced in Refs[4,5], we detect the singularities and determine their parameters without reference wave, by using a 1 mm-diam needle as an opaque strip, and observe at its shadow the Young's interference fringes produced by the edge diffraction waves from the needle rims[†]. In absence of singularity the Young's interference fringes are straight, with bright fringe at the center of shadow. The presence of optical vortex manifests itself by bending of interference fringes along the needle following the law:

$$\Delta\varphi(d,r) = m\left[\pi + \tan^{-1}(r/d)\right], \tag{3}$$

where m is the topological charge of an optical vortex, r is the height of the running point of the interference pattern over the vortex, and d is the needle half-width. Being referenceless, this technique can be applied for detecting and diagnostics of optical vortices not only in completely coherent monochromatic beams, but also in partially coherent[4,5] and polychromatic singular beams[3,9], where the use of separate reference wave is problematic. For that, the direction and magnitude of the fringe bending unambiguously determine, respectively, the sign of the topological charge and its modulus.

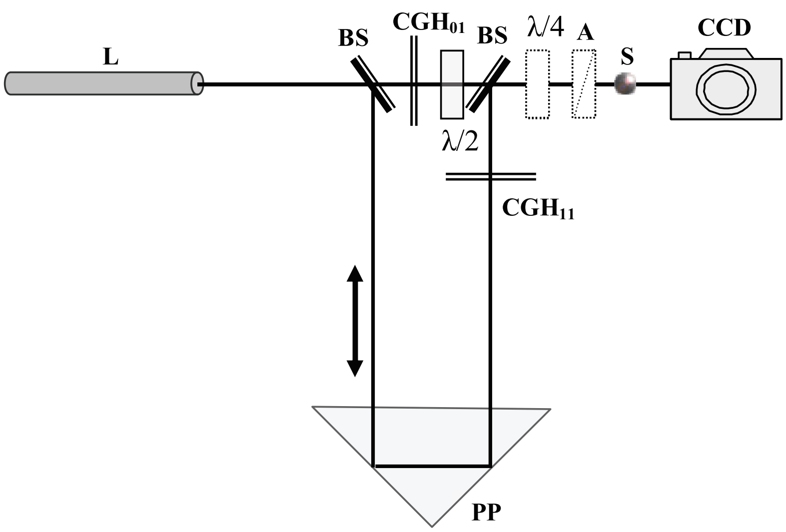


Fig. 1. Principal experimental arrangement for creating the combined singular beam and implementing reincarnation of optical singularities: *L* – laser; *BS* – beam-splitters; CGH_{01} and CGH_{11} – computer-generated holograms reconstructing Laguerre-Gaussian modes LG_0^1 and LG_1^1, respectively; *PP* - Porro prism; $\lambda/2$ - half-wave plate; $\lambda/4$ - quarter-wave plate; *A* – linear analyzer; *S* – opaque strip; *CCD* – CCD-camera.

Let us consider the case, when a path delay in the arrangement shown in Fig. 1 is much smaller than the preliminary measured coherence length of the laser used (5 cm against the coherence length about 27 cm). The fast axis of a half-wave plate coincides with the azimuth of polarization of the probing beam, so that two modes are of the same state of polarization. Coherent mixing of the modes results in disappearance of the ring singularity inherent in the mode LG_1^1 at the ring $\rho/w_z = 1$, where w_z is the width of the beam at the level e^{-1} from the maximal amplitude at the distance z from the caustics neck. At the same time, if two modes are in phase in the central part of the combined beam (for $\rho/w_z < 1$), they are of opposite phases at the ring $\rho/w_z = 1.45$. Being of the same intensity at this ring (see Fig. 2)

[†] This technique is discussed in more details in Ref. 10.

two partial modes interfere completely destructively and produce the ring singularity. The computed radial intensity distribution for this case is shown in Fig. 3. One can see very week side-lobe, whose intensity in practice may be lower than a noise, so that one observes the combined beam with the envelope resembling the mode LG_0^1 but narrowed, as it is seen in Fig. 4.

If the mentioned phase difference is changed, than the ring phase singularity disappears. Thus, such singularity is non-generic.

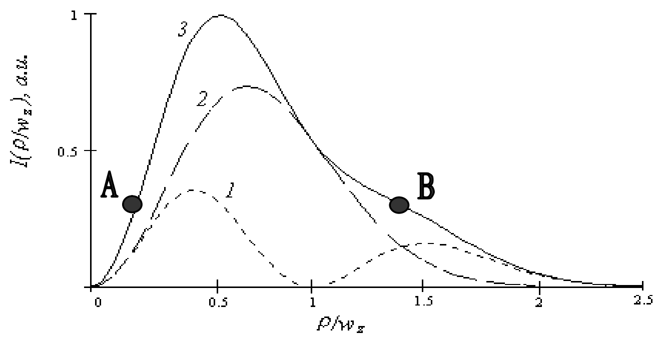


Fig. 2. Radial intensity distributions of isolated modes LG_1^1 (1) and LG_0^1 (2) for the power ratio 1:0.45, and of the combined singular beam (3) resulting from *incoherent* co-axial mixing of the modes.

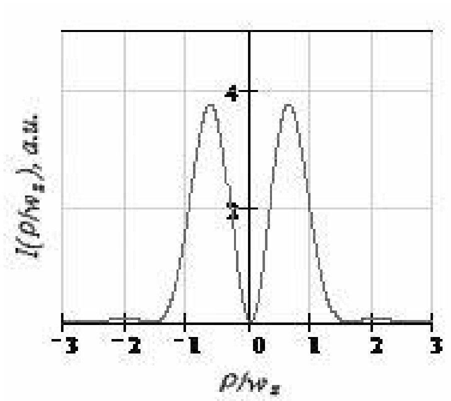


Fig. 3. Radial intensity distribution of the combined beam resulting from coherent co-axial mixing of the modes LG_0^1 and LG_1^1 that are in phase at the central part $\rho/w_z < 1$. Beside of the central vortex one observes the ring singularity at the ring $\rho/w_z = 1.45$.

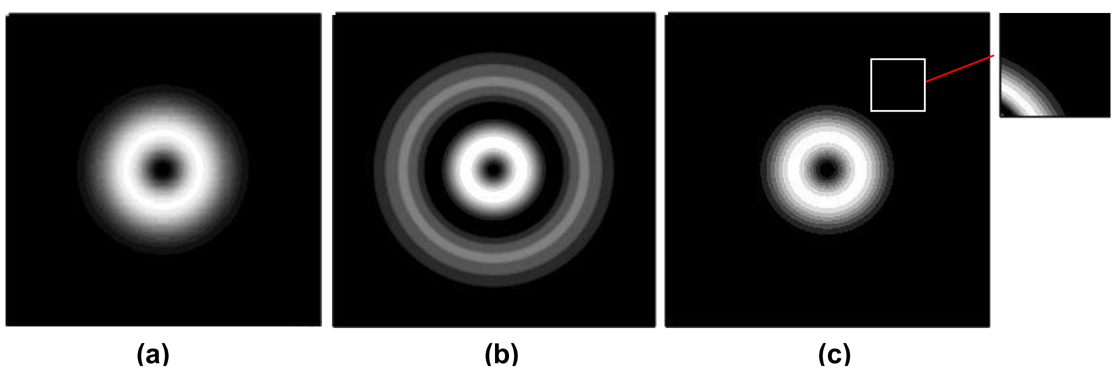


Fig. 4. Constituting modes LG_0^1 and LG_1^1, **a** and **b**, respectively, and the combined beam. Insert shows a weak side-lobe.

As the second step, let us now rotate a half-wave plate at 45°, so that two mutually coherent modes become orthogonally polarized. The ring phase singularity disappears, but if the phase difference between the modes is $\pi/2$, one can observe the ring polarization singularity such as $C-$contour. To visualize this singularity one must use a quarter-wave plate and linear analyzer in the combined beam in front of the testing needle. In this case one observes the pattern shown in Fig. 5. This polarization singularity is also non-generic and disappears when the phase difference between two modes changes. General solution of this problem for the orthogonally polarized modes belonging to arbitrary type of polarization is given in Ref. 6.

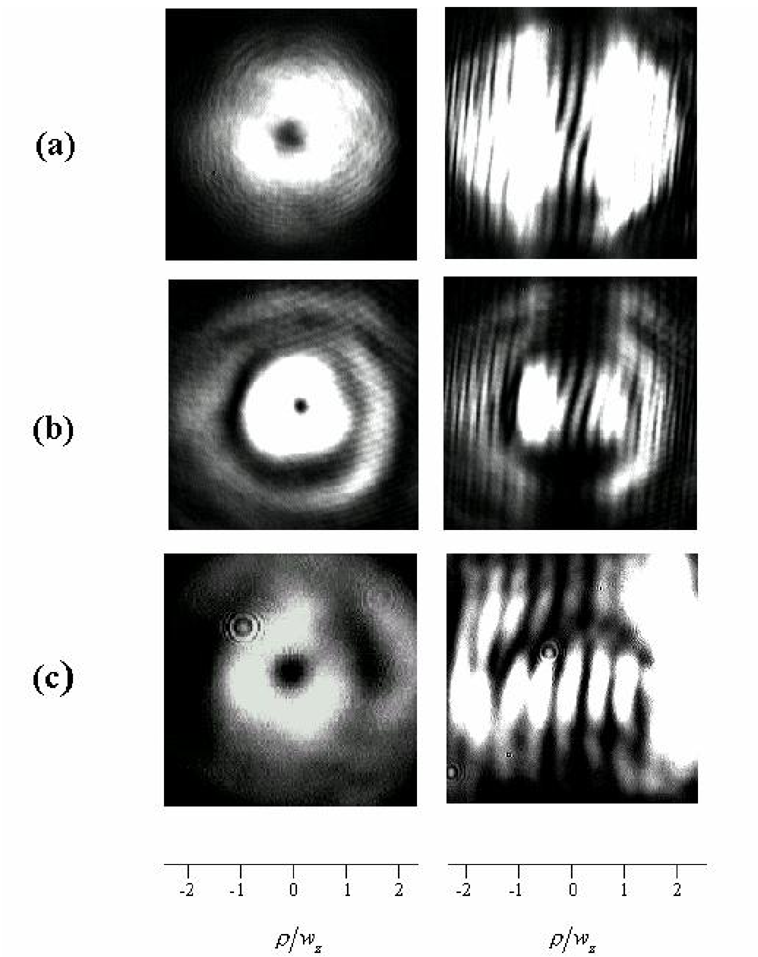


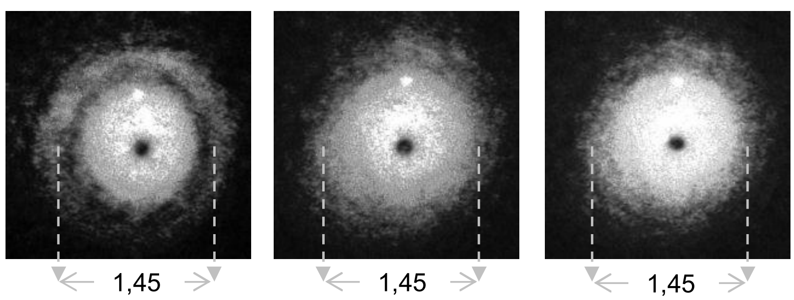
Fig. 5. **a** and **b** – orthogonally linearly polarized Laguerre-Gaussian modes LG_0^1 and LG_1^1, respectively (topological charge $m=-1$); **c** – combined beam for coherent superposition of these modes observed with a half-wave plate and linear analyzer; right column shows the result of diffraction diagnostics of the central vortex and ring polarization singularity at the ring $\rho/w_z = 1.45$.

As the third step, we provide the path delay in arrangement shown in Fig. 1 much larger than the coherence length of the laser (in our experiment, 80 cm against the coherence length about 27 cm). Orientation of a half-wave plate is the same as in the previous case, and mutually incoherent modes are orthogonally polarized. The combined beam is shown in Fig. 6. Performing the Stokes-polarimetric analysis of the combined beam, one can see that it is non-uniformly, partially polarized excluding the ring $\rho/w_z = 1$ where the beam is completely polarized with the polarization of the mode LG_0^1. At the same time, intensity of the combined beam behind the analyzer is constant for arbitrary orientations of the quarter-wave plate and linear analyzer at the ring $\rho/w_z = 1.45$, where the CDP $\mathscr{P}$ equals zero.

For mutually incoherent light beams the Stokes parameters are additive. So, for orthogonally linearly polarized modes LG_0^1 and LG_1^1 with the azimuths of polarization 0° and 90°, which is always possible owing to arbitrary choice of the coordinates, the CDP is calculated on the formula:

$$\mathscr{P}\left(d/w_z\right) = \frac{S_1^{(01)} - S_1^{(11)}}{S_0^{(01)} + S_0^{(11)}} \equiv s_1^{(01)} - s_1^{(11)} = \frac{I_{01} - I_{11}}{I_{01} + I_{11}}, \tag{4}$$

while $S_3^{(01)} = S_3^{(11)} = 0$ and polarization is linear. In Eq. (4) both the Stokes parameters and intensities of the modes are the functions of the dimensionless parameter ρ/w_z. Eq. (4) is justified everywhere excluding the central vortex, $\rho/w_z = 0$, where $\mathscr{P}$ is undetermined. These equation resembles the formula for the visibility of interference pattern. So, in the case under consideration the CDP $\mathscr{P}$ equals the difference of the normalized second Stokes parameters of the partial modes. One can see from Fig. 5 that polarization is everywhere *partial*, linear excepting the ring $\rho/w_z = 1$, where $\mathscr{P} = +1$ (completely polarized field with the state of polarization of the mode LG_0^1) ang the ring $\rho/w_z = 1.45$, where $\mathscr{P} = 0$ and the state of polarization is undetermined. Passing this ring the azimuth of polarization is changed by jump from 0° to 90°. So, the singularity of the CDP takes place at the ring $\rho/w_z = 1.45$, see Fig. 7.



(a) **(b)** **(c)**

Fig. 6. The combined singular beam consisting from mutually incoherent, orthogonally polarized modes LG_0^1 and LG_1^1 with the intensity ratio 1:0.45 with the ring singularity of the CDP at the ring $\rho/w_z = 1.45$ for three orientations of a linear analyzer: **a** – analyzer is matched with the mode LG_1^1, **b** – analyzer is matched with the mode LG_0^1, **c** – the axis of maximal transmittance of the analyzer is oriented at 45° to the azimuth of polarization of the partial modes; arrows show the ring where $\mathscr{P} = 0$.

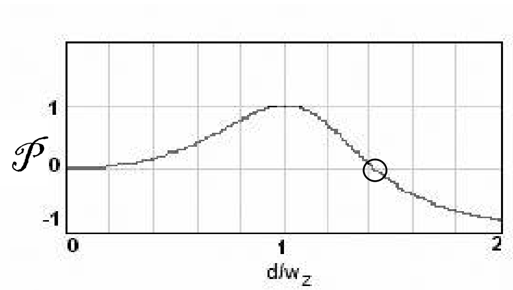


Fig. 7. Radial distribution of the CDP $\mathscr{P}$ for the combined beam consisting from mutually incoherent, orthogonally polarized modes LG_0^1 and LG_1^1 with the intensity ratio 1:0.45. A circle indicates the $U-$singularity position.

Important difference of such polarization singularity from previously discussed ones is that it is generic. Namely, the phase difference between two modes is undetermined by definition of incoherence. Thus, the strongest condition of the specific phase relations between two partial modes does not affect now the radial distribution of the states of polarization. Besides, if one changes the intensity ratio of two incoherently mixed modes (or such changes occur *per se* due to the difference in the Gouy phase for the modes LG_0^1 and LG_1^1 and, as a consequence, different diffraction spreading of such modes under propagation), the point in Fig. 7 where the CDP equals zero moves to the right or to the left, but never the singularity disappear in itself. It takes place always owing to crossing the graphs for the intensity distributions of incoherently mixed modes. In this meaning, the introduced singularity of the CDP is generic, and the combined beam occurs to be stable against environmental disturbances just owing to incoherent mixing of the partial modes. To all appearance, the nature of stability of optical singularities in nonlinear singular optics with partially coherent fields[11] is of the same nature.

It is clear that the same result, singularity of the CDP $\mathscr{P}$, can be obtained with the orthogonally polarized modes belonging to arbitrary type of polarization, *viz.* linear, circular or elliptical, with the corresponding changes in Eq. (4), which in the case of elliptical polarization is generalized to the form:

$$\mathscr{P}\left(d/w_z\right) = \sum_{i=1}^{3}\left(s_i^{(01)} - s_1^{(11)}\right). \qquad (5)$$

The last, fourth step consists in rotation of a half-wave plate at 45°, so that mutually incoherent modes become equally linearly polarized. In this case by the special choice of two probing beams at the radius of the combined beam one can experimentally reveal one more unusual singularity, such as singularity of the complex degree of cherence[12,13] (see also Refs. 3-5). For the mentioned intensity ratio of partial modes, the choice of the probing beams is as shown in Fig. 2, points *A* and *B*. Now, instead of an opaque strip as the device used for detecting the singularity, it is convenient to place in the combined beam the opaque screen with properly situated pinholes followed by a lens. Such arrangement is analogous to the Michelson's stellar interferometer used by Wolf and Thompson for experimental verification of the Van-Cittert – Zernike theorem[14,15]. Observation of an interference pattern is carried out at the back focal plane of the lens. For the mentioned choice of the probing points at the beam cross-section, four partial waves (from two modes for the each of two pinholes) are of the same amplitude. As a consequence, two separate interference patterns of the equal periods, the same mean intensities and the same visibilities, but shifted by a half-period are superimposed at the observation pane. So, at the center of pattern one observe the maximum for the mode LG_0^1 and the minimum for the mode LG_1^1, as it is shown in Fig. 8. Superposition of such interference patterns results in zero visibility that evidences on mutual incoherence of the waves at the probing points. Thus, we observe the two-point singularity of the complex degree of coherence, *viz.* zero magnitude of this value (zero visibility of interference fringes) and undetermined phase (localization of interference fringes)[5]. It is clear that such coherence singularity takes place also in the previous case, when two modes are polarized orthogonally. But the absence of interference fringes at the arrangement of the Michelson's stellar interferometer in the previous case follows both from mutual incoherency of the waves in two pinholes and from polarization orthogonality of partial waves. In contrast, in last case one can detect pure coherence singularity.

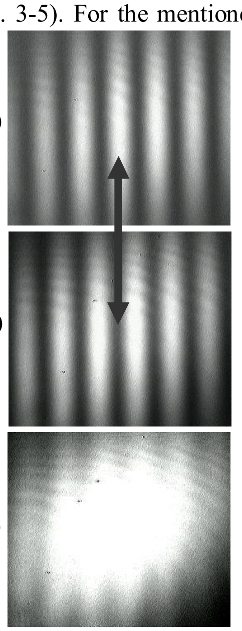


Fig. 8. Detecting the two-point singularity of the complex degree of coherence: **a** and **b** are partial interference fringes for two modes shifted by a half-period; **c** – resulting pattern with zero visibility.

4. CONCLUSIONS

In this paper we have demonstrated the chain of reincarnations of optical singularities, such as conventional phase singularities in homogeneously polarized, completely coherent combined vortex, non-generic polarization phase singularities in non-homogeneously polarized, completely coherent fields, the new type of polarization singularities, *viz.* singularity of the modified degree of polarization in non-homogeneously polarized, partially coherent fields, and the phase singularity of the complex degree of coherence in homogeneously polarized, equally polarized fields. For this purpose, we have used the interferometric arrangement with controllable path delay between two legs, and two weighted Laguerre-Gaussian modes, LG_0^1 and LG_1^1, in the legs of the interferometer with controllable states of polarization (equal or orthogonal).

It has been shown that both the singularity of the CDP in partially polarized, partially coherent combined beams and the coherence singularity in partially polarized or completely polarized combined beams are generic. Namely, such singularities are stable in respect to disturbances of the presumed intensity ratio of the partial modes, being, by definition of mutually incoherent modes, independent on the phase relations between these modes. Thus, singularity of the CDP takes place in the combined beam just due to crossing the curves corresponding to the radial intensity distributions for two constituting modes. Changing the intensity ratio of these modes results alone in shift of the ring where such singularity takes place along the radius of the combined singular beam, but not in destroying this singularity. This is in contrast with the non-generic polarization singularities in completely coherent combined beams, such as $C-$rings (or $L-$rings with the constant polarization azimuth for orthogonally circularly polarized partial modes), which are not topologically stable; such singularities take place only for specific amplitude ratio of the partial modes and phase difference between them and disappear by changing one or both mentioned parameters. It means that such singularities can be implemented in practice only with the limited experimental accuracy.

Thus, some types of optical singularities, *viz.* singularity of the CDP and singularity of the complex degree of coherence, are more stable under free propagation than polarization singularities in completely coherent combined beams. It explains higher stability of partially coherent combined beams in comparison with completely coherent ones and opens some feasibilities for using partially coherent and partially polarized singular beams in optical telecommunications. Now we aspire to extend the approach introduced here, in part the concept of the CDP and seeking for generic coherence and polarization singularities, on partially coherent random speckle fields with spatially non-uniform polarization.

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