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Vector singularities of the combined beams assembled from mutually incoherent orthogonally polarized components

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Abstract
It is shown that, for an incoherent superposition of the orthogonally polarized laser beams, the vector singularities of a specific type arise at the transversal cross section of a paraxial combined beam instead of common singularities, such as amplitude zeros or optical vortices (inherent in scalar, i.e. homogeneously polarized, fields), and C points, where polarization is circular, and L lines, along which polarization is linear (inherent in completely coherent vector, i.e. inhomogeneously polarized fields). There are U lines (closed or closing at infinity) along which the degree of polarization equals zero and the state of polarization is undetermined, and isolated P points where the degree of polarization equals unity and the state of polarization is determined by the non-vanishing component of the combined beam. U surfaces and P lines correspond to such singularities in three dimensions, by analogy with L surfaces and C lines in three-dimensional completely coherent vector fields. P lines directly reflect the snake-like distortions of a wavefront of the singular component of the combined beam. Crossing of the U line (surface) is accompanied by a step-like change of the state of polarization onto the orthogonal one. U and P singularities are adequately described in terms of the complex degree of polarization with the representation at the Stokes space, namely *at* and *inside* of the Poincaré sphere. The conditions of topological stability of U and P singularities are discussed, as well as the peculiarities of the spatial distribution of the degree of polarization in the closest vicinity to such singularities. Experimental examples of reconstruction of the combined beam's vector skeleton formed by U and P singularities as the extrema of the complex degree of polarization are given. Comparison with the related investigations is provided.

Keywords: optical singularities, partial polarization, partial coherence, Stokes polarimetry

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The central subject of singular optics of completely coherent but inhomogeneously polarized fields resulting, for example, from stationary multiple scattering of laser radiation is C points (the points where a field is circularly polarized and the azimuth of polarization is undetermined) and L lines (the lines along which polarization is linear with smoothly changing azimuth of polarization and the direction of rotation of the electrical vector—handedness—is undetermined) at the transversal cross section of a beam. Snake-like C lines and L surfaces correspond to such singularities in three dimensions. The set of C points and L lines obeys well-known sign principles and forms the vector skeleton of a coherent, inhomogeneously polarized beam [1–5], namely knowing the characteristics of the field at C points and L lines, one can predict in a qualitative manner the field behavior (in part, changing the state of polarization) at other areas of the beam. This property of the polarization singularities follows from their *genericity* [6], i.e. structural stability with respect to small perturbations of the initial conditions or perturbations of a freely propagating beam. Nongeneric polarization singularities resulting from coherent mixing of weighed orthogonally polarized Laguerre–Gaussian (LG) modes with different radial indices have also been elaborated [7]. The universal theoretical and experimental approach for the investigation of vector singularities is based on determining the Stokes parameters as a function of spatial coordinates at the analyzed transversal cross section of a beam, followed by determining the spatial distributions of the azimuth of polarization and the angle of ellipticity and identification of singular elements of a field.

A new problem arises in the case of *incoherent* mixing of orthogonally polarized beams, at least one of which possesses phase singularities (optical vortices [8]), i.e. the points at the transversal cross section of a beam where the field amplitude vanishes and its phase is undetermined. Passing such points the phase of a field is step-like changed by π. For incoherent coaxial mixing of such a beam with the orthogonally polarized plane wave (or with another orthogonally polarized vortex beam) the usual singularities, such as optical vortices as well as C points and L lines, are absent in the combined beam. It follows from the fact that optical singularities considered within the framework of different models of optical phenomena are fundamentally different. In paper [9] this circumstance is illustrated, for example, for the sequence: caustic singularities of geometrical optics–phase singularities (optical vortices) of scalar wave optics–polarization singularities of vector singular optics–quantum vacuum of quantum optics. For a transition at a higher level of description of a field the singularities present at lower levels disappear, and new singularities arise. For that, there is no direct interconnection between singularities of different kinds which would provide a transformation of some kind of singularity into some other one by changing one experimental parameter. A significant example of this statement is the phase singularities of spatial correlation functions and the complex degree of coherence at partially coherent scalar (homogeneously polarized) light fields where common phase singularities of the complex amplitude of a field are absent [10–13].

A less investigated case is when a field is partially coherent and, at the same time, inhomogeneously polarized. Some examples of such fields were elaborated on in the papers [14, 15]. It has been shown in [14] that the state of polarization of a not strictly monochromatic field is described by Lissajous figures rather than by the conventional polarization ellipse. The study [15] is devoted to interpretation of the distribution of the states of (partial) polarization of clear-sky daylight from the singular optics point of view. It is shown, in part, that the points with zero degree of polarization and undetermined state of polarization exist in the 'sea' of linear polarizations with changing azimuth.

It has been recently shown [16, 17] that, at the transversal cross section of the combined beams resulting from incoherent coaxial superposition of orthogonally polarized singular laser beams, instead of C points and L lines, the vector singularities of another type arise, namely, U (*unpolarized*) contours (closed or closing at infinity), along which the degree of polarization equals zero and the state of polarization is undetermined, and P (completely *polarized*) points where the degree of polarization reaches its maximal (unity) magnitude and the state of polarization corresponds to the non-vanishing component of the combined beam. U surfaces and P lines correspond to such singularities in three dimensions, like L surfaces and C lines in three-dimensional completely coherent vector fields. Moreover, P lines directly reflect so-called snake-like distortions of a wavefront [18] corresponding to optical vortices. In this paper we continue to investigate the properties of U and P singularities.

This paper is organized as follows. Following the approach developed in paper [17], in section 2 we build the complex parameter, namely the complex degree of polarization, which is used in section 3 for representation of U and P singularities at the Stokes space and discuss the structural stability of such singularities in real three-dimensional space. In section 4 peculiarities of the distribution of the degree of polarization in the vicinity of U and P singularities are considered. The experimental technique and the results on the reconstruction of such singularities and the vector skeleton of the combined beams are presented in section 5. Section 6 summarizes our main new findings.

2. Complex degree of polarization

Conventionally, the degree of polarization of light is defined as the real, non-negative value [19–23]:

$$P = \frac{I_{\mathrm{p}}}{I_{\mathrm{p}} + I_{\mathrm{u}}}; \qquad 0 \leqslant P \leqslant 1, \tag{1}$$

where $I_{\mathrm{p}}, I_{\mathrm{u}}$—intensities of completely polarized and completely unpolarized components of a beam, respectively. Note that there is no polarization device providing the decomposition of a beam into such components. Alternatively, the polarization degree can be represented through the invariants (determinant and spur) of Wolf's coherency matrix: $\{\mathbf{J}\}$ [22]:

$$P = \sqrt{1 - 4\det\{\mathbf{J}\}/\mathrm{Sp}^2\mathbf{J}}, \tag{2}$$

(the completely polarized limit corresponds to $\det\{\mathbf{J}\} = 0$, while the completely unpolarized limit corresponds to $J_{xx} = J_{yy}$ and $J_{xy} = J_{yx} = 0$). Though the non-diagonal elements of the coherency matrix are not directly measured, they are connected with the Stokes parameters which can be determined from measurements of six intensities: $S_0 = I_0 + I_{90}$, $S_1 = I_0 - I_{90}$, $S_2 = I_{+45} - I_{-45}$ and $S_3 = I_{\mathrm{r}} - I_{\mathrm{l}}$. The degree of polarization can also be defined in the terms of the Stokes parameters:

$$P = \sqrt{s_1^2 + s_2^2 + s_3^2}, \tag{3}$$

where $s_i = S_i/S_0$, $(i = 1, 2, 3)$—the normalized second, third and fourth Stokes parameters. At inhomogeneously polarized fields all Stokes parameters are functions of the spatial coordinates. So, the definition (3), in contrast with (1) and (2), assigns the experimental procedure for determining the degree of polarization for light consisting of arbitrarily polarized components. It is important in this study that the second, third and fourth Stokes parameters relate to the representation of a polarized light at the Poincaré sphere, being simply the Cartesian coordinates of the point imaging some state of polarization.

The Stokes parameters can be both positive and negative. So, the normalized second, third and fourth Stokes parameters of the orthogonally polarized fields are related as $\{s_1, s_2, s_3\}$ and $\{-s_1, -s_2, -s_3\}$. Thus, the Stokes parameters contain information on the specific state of polarization of a field: the azimuth of polarization, $\alpha = 0.5\tan^{-1}(s_2/s_1)$ $(-\pi/2 \leqslant \alpha < \pi/2)$, and the ellipticity angle, $\beta = 0.5\sin^{-1} s_3$ $(-\pi/4 \leqslant \beta \leqslant \pi/4)$. However, this information is lost when one defines the degree of polarization of a beam using the quadratic values (3). Here we show that the concept of the degree of polarization can be generalized in such a manner that the new definition will contain complete information both on the degree (P) and the state (α, β) of polarization.

Let us use the concepts developed in [23] to introduce the complex degree of polarization (CDP). In the cases when the amplitude and vibrational (initial) phase of a beam are not relevant and only the state of polarization is of interest, the beam can be described by the so-called circular complex polarization variable:

$$\chi_{r,l} = E_r/E_l = (|E_r|/|E_l|)\exp(\delta_r - \delta_l), \tag{4}$$

where E_r and E_l—the components of the circular Jones vector. The variable $\chi_{r,l}$ is the function of two real arguments, namely the amplitude ratio of right-hand and left-hand circularly polarized components of a beam, $|E_r|/|E_l|$, and the phase difference between them, $(\delta_r - \delta_l)$. The use of the circular complex polarization variable instead of the Cartesian one derived from the conventional Cartesian Jones vector is preferable while the decomposition of a beam into circular components is unambiguous, i.e. it is not dependent on the choice of the coordinate frame (azimuth). This variable can be rewritten in terms of the azimuth of polarization and the ellipticity angle:

$$\chi_{r,l} = \tan(\beta + \pi/4)\exp(-i2\alpha). \tag{5}$$

The circular polarization variable uniquely determines the state of polarization of completely (elliptically, in the general case) polarized beam at the circular complex plane [23] and, through stereographic projection, at the Poincaré sphere.

Let us introduce the CDP by the definition

$$\mathscr{P} = P\mathscr{N}\chi_{r,l}, \tag{6}$$

where $\mathscr{N} = |\chi_{r,l}|^{-1} = |E_l|/|E_r|$ is the normalizing factor matching the dimensions of the infinite circular complex plane and the Poincaré sphere of unit radius. One can put in the correspondence to the CDP the polarization vector at the Stokes space, $\mathbf{s} = s_1\mathbf{i} + s_2\mathbf{j} + s_3\mathbf{k}$ $(|\mathbf{s}| = P)$, as is shown in figure 1. It is remarkable that the use of the Stokes space (within the Poincaré sphere) provides imaging

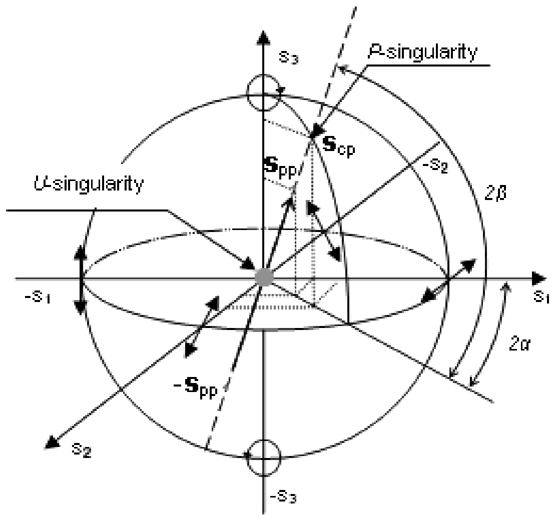

Figure 1. Representation of completely and partially polarized beams at the Stokes space: the points at the Poincaré sphere image completely polarized field (P singularities of the combined beams); points inside the sphere—partially polarized fields; the origin of coordinates—completely unpolarized field. $\mathbf{s}_{cp}$ and $\mathbf{s}_{pp}$ are the polarization vectors of completely polarized ($P = 1$) and partially polarized ($P = 0.64$) fields, respectively. Transition from $\mathbf{s}_{pp}$ to $-\mathbf{s}_{pp}$ goes on through the U singularity.

not only completely polarized beams (*at* the sphere, $|\mathbf{s}| = 1$), which correspond to 'pure' states of a field in the sense of quantum optics and statistical electrodynamics [19, 21], but also partially polarized beams (*inside* the sphere, $|\mathbf{s}| < 1$), which correspond to 'mixed' states of a field; the center of the Poincaré sphere, $|\mathbf{s}| = 0$, corresponds to the zero degree of polarization: $s_1 = s_2 = s_3 = 0$, see figure 1. Points outside the sphere do not represent any state of polarization. The CDP defined in such a way comprehensively characterizes both the state and the conventional degree of polarization, for $-1 \leqslant \mathscr{P} \leqslant 1$, $|\mathscr{P}| = P$, as will be illustrated in section 3. Note that, as introduced by us. the CDP differs from the complex degree of *mutual* polarization (CDMP) [24], that is two-point characteristics of an inhomogeneously polarized field.

3. Representation of *U* and *P* singularities at the Stokes space

Let us apply the concept of the CDP for elaborating the case of incoherent coaxial mixing of two orthogonally polarized beams, at least one of which is inhomogeneous in intensity at the transversal cross section, and mean intensities are commensurable. The paraxial (beam-like) approximation is assumed hereinafter. The following consideration is quite general, being well founded for arbitrary combinations of the orthogonally polarized partial beams, irrespective of the type of polarization (linear, circular or elliptical).

Along the lines at the transversal cross section of the combined beam where intensities of partial orthogonally polarized beams are equal, the real degree of polarization (3) is zero and the state of polarization is undetermined (U singularities, the center of the Poincaré sphere). Crossing such lines at the transversal cross section of the beam (or associated surfaces at the three-dimensional field) is accompanied with a step-like change of the state of polarization onto the *orthogonal* one. It corresponds to the change of the direction of the polarization vector at the Stokes space onto the *opposite* one, see figure 1, as follows from the rule of measurement of the angles (2α, 2β) at this space. Moving on the transversal cross section of the combined beam corresponds to walking of the imaging point at the Stokes space along the diameter of the Poincaré sphere that connects two orthogonal polarization states, rather than walking at the sphere, as in the case of coherent vector singular optics.

Note that, in the case when two constituting orthogonally polarized beams are partially coherent, U singularities do not appear. At the lines where the intensities of these beams are equal the field occurs to be partially polarized, $0 < P < 1$. Crossing of such lines corresponds to moving of the imaging point at the Stokes space along some complex trajectory inside the Poincaré sphere, which is determined by both the degree of mutual coherence and the phase difference of the partial beams, rather than along the diameter of the Poincaré sphere, avoiding the center of the sphere (U singularity). As mutual coherence of the partial beams increases, this trajectory approaches the sphere which corresponds to increasing the degree of polarization of the combined beam.

It must be emphasized that the point-like U singularities at the transversal cross section of a beam of the considered type are not topologically stable. This is one of the important differences between U singularities considered here and the points with zero degree of polarization discussed in paper [15]. So, for the changing intensity of any partial beam by an infinitely small value, $I \to I \pm \delta I$, U points annihilate or degenerate into closed U contours. This statement is illustrated in figure 2 for the example of incoherent coaxial mixing of the mode LG11 with a plane wave whose intensity (approximately) equals the intensity of the side-lobe maximum of the LG11 mode. Only U contours at the slopes of the first maximum of the LG11 mode are stable. Really, small changes of the LG mode-to-reference intensity ratio (far from the phase transition threshold, when the intensity of a reference wave exceeds the peak intensity of the LG11 mode) results only in changing the size of U contours. As a consequence, the seeming detection of point-like U singularities must be considered as the result of limited accuracy of an experiment. The same is true for *crossings* of two U contours [17]. Note, such properties are quite analogous to the topological properties of L lines into coherent inhomogeneously polarized fields with vector singularities [25].

If one of the incoherently mixed orthogonally polarized beams contains optical vortices, then the field at these points is completely polarized, with the state of polarization of the non-vanishing component of the combined beam. Such points of a beam are imaged *at* the Poincaré sphere and correspond to P singularities. It has been shown [17] that changing the intensity ratio of the partial beams results in changing sizes, form and number of the U contours up to the complete disappearance of them when the intensity of the non-singular component exceeds the maximal peak intensity of the singular

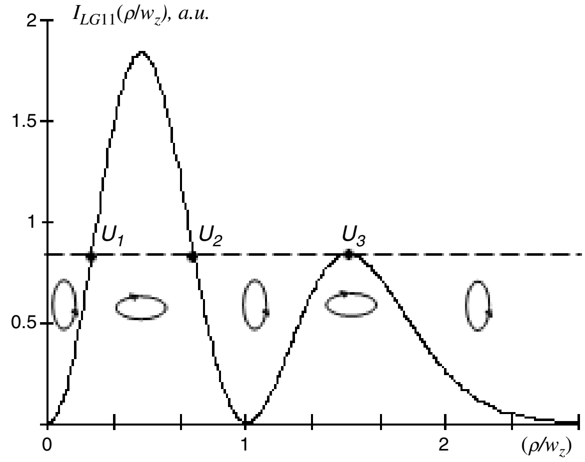

Figure 2. U singularities at incoherent superposition of orthogonally elliptically polarized LG11 mode (solid line) and plane wave (dashed line). Singularities U_1 and U_2 far from the peak intensity are structurally stable, while singularity U_3 is not stable, being strongly dependent on the intensity ratio of the partial beams.

one. At the same time, the number and positions of P points are unchanged. P points, in contrast to U contours, also exist in the case of partial coherence of constituting beams.

At last, if both partial beams contain optical vortices (as in the case of two orthogonally polarized speckle fields), then the set of U singularities is added by two sets of P points of opposite signs corresponding to the orthogonal states of the polarization of partial beams. Such P points are imaged by the ends of the diameter of the Poincaré sphere. In this case, the real degree of polarization for each set of P points equals unity, and $\mathscr{P}$ differ in signs.

Let us emphasize once more that the above-mentioned is true not only for orthogonally linearly polarized beams but also for pairs of orthogonally polarized beams belonging to arbitrary type of polarization, including circular and elliptical.

Both the real degree of polarization and the azimuth of polarization and the ellipticity angle are represented in terms of the Stokes parameters. Besides, the same values as for determining the fourth Stokes parameter are sufficient for determining the normalizing factor $\mathscr{N}$ in equation (6). As a result, the most general definition of the CDP for orthogonally elliptically polarized partial beams can be written in the form

$$\mathscr{P} = \sqrt{s_1^2 + s_2^2 + s_3^2} \cdot \sqrt{I_l/I_r} \times [\tan(0.5 \sin^{-1} s_3 + \pi/4) \exp(-\mathrm{i} \tan^{-1} s_2/s_1)]. \qquad (7)$$

In partial cases of linearly or circularly polarized partial beams this general definition is simplified. So, for the case of circularly polarized components ($s_1 = s_2 = 0$) equation (7) is rewritten in the form

$$\mathscr{P} = |s_3| \cdot \sqrt{I_l/I_r} \cdot [\tan(0.5 \sin^{-1} s_3 + \pi/4)], \qquad (8.1)$$

while for the case of linearly polarized components ($s_3 = 0$) one obtains

$$\mathscr{P} = \sqrt{s_1^2 + s_2^2} \cdot \sqrt{I_l/I_r} \cdot [\exp(-\mathrm{i} \tan^{-1} s_2/s_1)]. \qquad (8.2)$$

So, for example, when $\alpha = \pi/12$, $\beta = 0$, $P = 0.7$ and $\mathcal{N} = |E_l|/|E_r| = 1$ (sign of linear polarization), equation (8.2) takes the form: $\mathscr{P} = 0.7\exp(-i\,0.5235)$, so that the phase of the CDP is simply the doubled azimuth of the polarization of a beam.

Thus, two-dimensional Stokes polarimetric analysis of the beams combined from mutually incoherent orthogonally polarized components and finding out the extrema of the CDP is an adequate technique for determining the positions of U and P singularities and reconstruction of the vector skeletons of such beams.

4. Distribution of the degree of polarization in the vicinities of *U* and *P* singularities in real space

Each optical singularity can be considered as some local structure with point or linear core with undetermined (singular) magnitudes of some parameter for the transition of a *smoothly* changing control parameter of the system through the threshold value. Such parameters are different for different types of singularities. As was mentioned above, the azimuth of polarization is singular and the angle of ellipticity is the control parameter at the C point at a completely coherent inhomogeneously polarized field. In contrast, handedness is undetermined, while the azimuth of polarization is the smoothly changing control parameter at the L line. At the areas between C points and L lines the state of polarization smoothly changes when the degree of polarization is unity.

The case considered by us is essentially different. U singularities separate the areas with constant (orthogonal) states of polarization, namely the magnitudes of ellipsometric parameters α and β; only the degree of polarization changes from point to point. This case differs from the one considered in [15], where both the degree and the state (azimuth) of linear polarization change from point to point.

Let us emphasize the important peculiarity of the distribution of the degree of polarization in the vicinity of U and P singularities, which is also shared by other kinds of singularities. The typical conical structure of the vicinity of amplitude zeros for several examples is presented in figure 3. The conical graph of an amplitude of common harmonic distribution (figure 3(a)) at amplitude zero is undifferentiable; undifferentiability disappears in the corresponding spatial intensity distribution. The same is observed for amplitude and intensity distributions of a mode LG01 (figure 3(b)). Passing from the complex degree of coherence to its modulo (figure 3(c)) results in a peg-shaped vicinity of the phase singularity of this value [11]. One can see from figure 3(d) that the same takes place when one passes from the complex degree of polarization $\mathscr{P}$ to the real value P. The conical structure of the distribution of the degree of polarization (in contrast to the field intensity distribution) takes pace also in the vicinities of P points.

The conical structure of the distribution of the degree of polarization near U and P singularities enables us to use the reference wave incoherent and orthogonally polarized

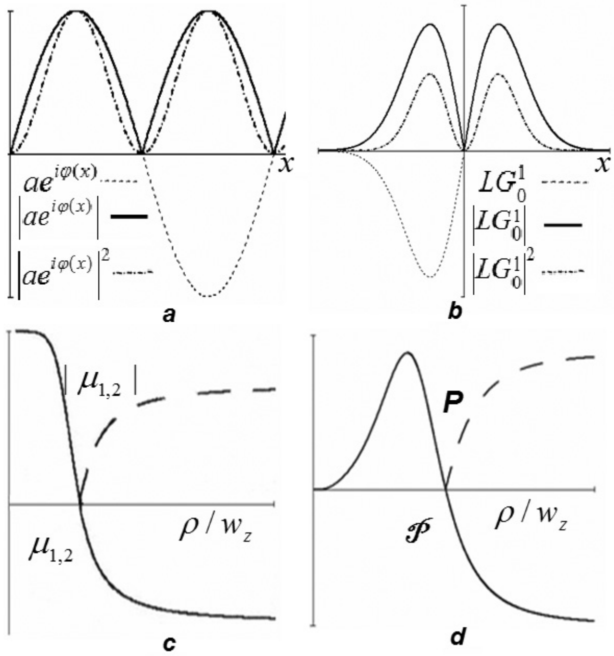

Figure 3. Conical vicinity of optical singularities for the modulo of complex amplitude of a harmonic signal (a), for the LG01 mode (b), for the modulo of the complex degree of coherence of the scalar combined beam (c) and for the degree of polarization (d). At left branches of the fragments (c) and (d) dashed and solid curves coincide.

with respect to the object wave with optical vortices for determining the vortex positions. This is an alternative to the conventional interference technique [18] whose spatial resolution is not high, being determined by the period of interference fringes. The polarimetric technique provides much higher spatial resolution, which is of importance for diagnostics of closely positioned vortices and differentiation of them from the vortices with multiple topological charges. In fact, the cone generatrix serves as the 'pointer' of singularities of the considered kind. For that, even accounting for optical noise, noise of detecting and computer processing of experimental data, one can expect spatial resolution at the level of dozens of micrometers (for typical linear sizes of pixels of a CCD camera: 4–5 μm).

The use of the orthogonally polarized reference beam for determining the positions of optical vortices at scalar statistical fields was proposed earlier [26] and has recently been implemented [27]. In [27], however, the reference and object waves were coherently mixed and the Stokes polarimetry was applied for determining the phase difference between the orthogonally *linearly* polarized components of the elliptically polarized combined beam, from which the positions of optical vortices can be determined. The use of a reference wave mutually incoherent with the singular one, providing the same spatial resolution, does not presume adjusting an optical arrangement with interferometric accuracy and is not sensitive to fluctuations of a phase difference of two beams, in part, due to vibrations.

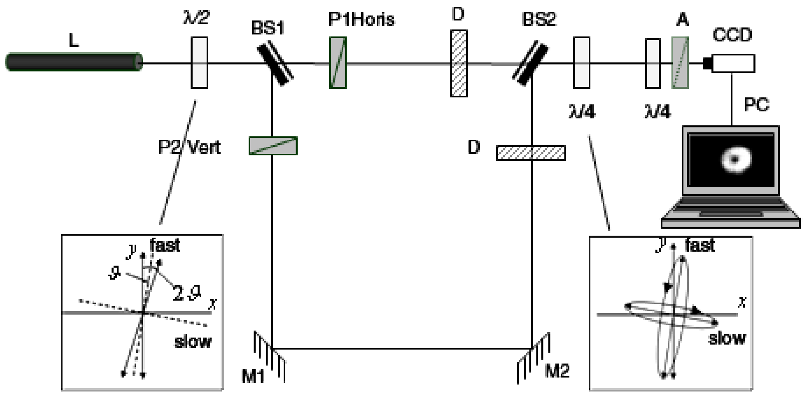

Figure 4. Experimental arrangement: L—laser; $\lambda/2$ and $\lambda/4$—half-wave and quarter-wave plates, respectively; BS1, BS2—beamsplitters; P1, P2—polarizers; M1, M2—mirrors; D—diffusers; A—linear analyzer; CCD—CCD camera; PC—personal computer. Action of a half-wave plate and a quarter-wave plate at the input and output of the interferometer, respectively, is shown in insets.

5. Experimental determining the positions of *U* and *P* singularities and reconstruction of vector skeleton of the combined beams

The experimental arrangement for determination of the positions of U and P singularities and reconstruction of the vector skeleton of partially coherent inhomogeneously polarized combined beams is shown in figure 4. The beam of the He–Ne laser is divided into two partial beams in the interferometer. One sets the path delay at the lower leg of an interferometer considerably exceeding (in our experiments, approximately by three times) the coherence length of the laser used.

Two samples of ground glass placed at the legs of the interferometer generate scalar fields supporting optical vortices. The parameters of surface roughness of the samples provide complete destruction of the regular components at the scattered radiation and formation of the developed speckle fields. In the experiment demonstrated below, we use samples with approximately equal sizes, which leads to the equal mean size of speckles in two partial beams, and sets the ratio of average intensities of the two beams close to unity.

A half-wave plate at the interferometer input serves for fine control of the intensity ratio at the legs with the constant resulting intensity of radiation at the interferometer output. Polarizers at the legs of the interferometer set the orthogonal states of linear polarization (horizontal and vertical). A quarter-wave plate at the interferometer output, depending on its orientation, transforms polarizations of the mixed beams into orthogonal elliptical or circular ones, see figure 4. Note that *any* phase plate, not only a quarter-wave one, acts similarly: depending on the orientation it changes the states of polarization of two beams (generally, to elliptical type) without violating their orthogonality. Of course, only using a quarter-wave plate provides implementation of the pair of orthogonal circular polarizations. So, two quarter-wave plates at the legs of an interferometer, as is used conventionally, may be replaced by a single plate at the output, which automatically (without the necessity of the fine adjustment of the two plates inside the interferometer) provides arbitrary pairs of orthogonally polarized beams. Another important advantage of such placing of a quarter-wave plate is that the distortions of the state of polarization of a beam at the lower leg of an interferometer due to the difference of the Fresnel reflection coefficients for p- and s-components for mirrors and a beamsplitter, which may result in non-orthogonality of two beams at the interferometer output, are excluded. In the experiment demonstrated below the angle between the fast axis of a quarter-wave plane and the azimuth of polarization of the horizontally polarized beam was 15°. For that, this beam takes the following ellipsometric parameters: $\alpha = \beta = 15°$; the normalized second, third and fourth Stokes parameters are $\{3/4; \sqrt{3}/4; 1/2\}$. The vertically polarized beam is transformed into an elliptically polarized one with the following parameters: $\alpha = \beta = -75°$, and the normalized second, third and fourth Stokes parameters are $\{-3/4; -\sqrt{3}/4; -1/2\}$. A quarter-wave plate and a linear analyzer in front of the CCD camera matched with the personal computer serve for registration of the coordinate intensity distributions needed for determining the Stokes parameters and the CDP.

The typical result of reconstruction of U and P singularities is shown in figure 5. The areas with different colors at the right fragment correspond to the orthogonal states of polarization separated by U contours; two sets of P points of opposite signs are depicted by red and blue asterisks. In contrast with [17], where P points arise for incoherent superposition of the speckle field with a plane wave only at one set of areas corresponding to the state of polarization of the reference wave, here one identifies P points at each set of areas. As was expected, squares of red and blue areas are approximately equal due to equal scales of speckle patterns. It was observed that, similarly to the model of combined beams investigated in [17], namely LG mode with the orthogonally polarized plane reference wave, changing the intensity ratio of the partial beams leads to changing the form and sizes of the U contours, as well as changing the square ratio of the areas corresponding to the predominance of one of the orthogonal states of polarization. However, the positions of P points remain unchanged, being determined by the positions of the component vortices.

The three-dimensional spatial distribution of the CDP reconstructed from the experimental data is shown in figure 6,

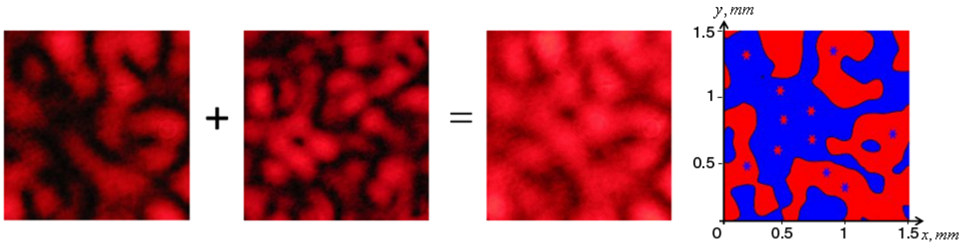

Figure 5. Fragments of mutually incoherent, orthogonally polarized speckle fields and the combined beam; experimentally reconstructed U contours separating the areas with the orthogonal states of polarization depicted by different colors, and P points of the opposite signs depicted by asterisks (right fragment) form the vector skeleton of the combined beam.

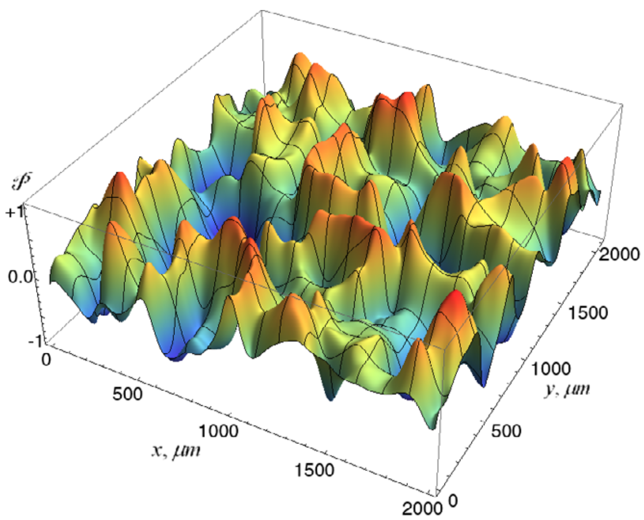

Figure 6. Three-dimensional spatial distribution of the complex degree of polarization $\mathscr{P}$ at the transversal cross section of the combined beam reconstructed from experimental data.

where one clearly identifies P singularities of opposite signs (plus one at the vertical axis, upwards, and minus one, downwards) with respect to a mean line (zero, U singularities) with characteristic conical vicinities.

One can get information about the distribution of the complex degree of polarization from this figure but not about the specific state of polarization. Information about the state of polarization is obtained from measured Stokes parameters. The only conclusion about the state of polarization that can be obtained from figure 6 is that above and below zero (at vertical axis) polarization states are orthogonal to each other.

6. Conclusions

Incoherent superposition of orthogonally polarized laser beams, at least one of which contains optical vortices, results in a partially spatially coherent and partially polarized combined beam. The degree of polarization is the function of spatial coordinates. Only two orthogonal states of polarization take place at the transversal cross section of the beam, and the areas with such states of polarization are separated by U singularities, i.e. the lines at which the degree of polarization equals zero. Crossing U lines is accompanied by a step-like change of the sign of the CDP (at the Stokes space) and, respectively, by a step-like change of the state of polarization into an orthogonal one. At points where one of the orthogonally polarized components undergoes phase singularity, the degree of polarization reaches a magnitude of unity (P singularity), and the state of polarization is determined by the non-vanishing component of the combined beam. U surfaces and P lines correspond to such singularities in three-dimensional space.

Two-dimensional Stokes polarimetric analysis of the combined beams of this kind provides experimental determination of the positions of U lines and P points and, in such a way, reconstruction of the vector skeleton of the beam. The distribution of the degree of polarization has a specific conical structure in the vicinity of the extrema of this value. Revealing such a structure, in part in the vicinities of P points, enables us to determine the positions of optical vortices at scalar (homogeneously polarized) fields using coaxial, mutually incoherent and orthogonally polarized reference waves instead of the conventional off-axis or on-axis interference technique or the technique based on the use of a mutually coherent orthogonally polarized reference wave.

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References

[1] Nye J F and Hajnal J V 1987 *Proc. R. Soc.* A **409** 21
[2] Berry M V and Dennis M R 2001 *Proc. R. Soc.* A **457** 141
[3] Freund I 2002 *Opt. Commun.* **201** 251
[4] Freund I, Soskin M S and Mokhun A I 2002 *Opt. Commun.* **208** 223
[5] Soskin M S, Denisenko V G and Egorov R I 2004 *J. Opt. A: Pure Appl. Opt.* **6** S281
[6] Nye J F 1999 *Natural Focusing and Fine Structure of Light* (Bristol: Institute of Physics Publishing)
[7] Bogatyryova H V, Felde Ch V, Polyanskii P V and Soskin M S 2004 *Opt. Spectrosc.* **97** 833
[8] Soskin M S and Vasnetsov M V 2001 *Prog. Opt.* **42** 219
[9] Berry M V and Dennis M R 2004 *J. Opt. A: Pure Appl. Opt.* **6** S178
[10] Ponomarenko S A 2001 *J. Opt. Soc. Am.* **18** 150

[11] Bogatyryova G V, Felde Ch V, Polyanskii P V, Ponomarenko S A, Soskin M S and Wolf E 2003 *Opt. Lett.* **28** 878

[12] Gbur G and Visser T 2003 *Opt. Commun.* **222** 117

[13] Palacios D M, Maleev I D, Marathay A S and Swartzlander G A Jr 2004 *Phys. Rev. Lett.* **92** 143905

[14] Freund I 2003 *Opt. Commun.* **226** 351

[15] Berry M V, Dennis M R and Lee R L 2004 *New J. Phys.* **6** 162

[16] Felde Ch V, Bogatyryova H V, Polyanskii P V and Soskin M S 2008 *Proc. SPIE* **7008** 1–8

[17] Felde Ch V, Chernyshov A A, Bogatyryova G V, Polyanskii P V and Soskin M S 2008 *JETP Lett.* **88** 418

[18] Baranova N B, Mamaev A V, Pilipetskii H F, Shkunov V V and Zel'dovich B Ya 1983 *J. Opt. Soc. Am.* **73** 525

[19] Shurcliff W A 1962 *Polarized Light: Production and Use* (Cambridge: Harvard University Press)

[20] O'Neil E L 1963 *Introduction to Statistical Optics* (Massachusetts: Addison-Wesley)

[21] Klauder J R and Sudarshan E C G 1968 *Fundamentals of Quantum Optics* (New York: Benjamin)

[22] Born M and Wolf E 1997 *Principles of Optics* 7th edn (Cambridge: Cambridge University Press)

[23] Azzam R M A and Bashara N M 1977 *Ellipsometry and Polarized Light* (Amsterdam: North-Holland)

[24] Ellis J and Dogariu A 2004 *Opt. Lett.* **29** 536

[25] Vasil'ev V and Soskin M 2008 *Opt. Commun.* **281** 5527

[26] Freund I 2001 *Opt. Lett.* **26** 1996

[27] Denisenko V G, Minovich A, Desyatnikov A S, Krolikowski W, Soskin M S and Kivshar Yu S 2008 *Opt. Lett.* **33** 89